Schedule: Algebraic Vector Bundles

Thursday, June 13, 2002

Organizers: V. Ancona, N.M. Kumar, G. Ottaviani, C. Peterson, A.P. Rao

9:00 - 9:30 Jean Vallès *Jumping lines of Logarithmic bundles on* $\mathbf{P}^2$

9:40 - 10:10 Vikram Mehta *Harder-Narasimhan Filtration of Principal Bundles*

Break

10:45 - 11:15 Edoardo Ballico *Holomorphic vector bundles*

11:20 - 11:50 Günther Trautmann *The Moduli Problem for Instanton Bundles*

11:55 - 12:25 Luca Chiantini *Vector bundles and curves on hypersurfaces of* $\mathbf{P}^4$

12:30 - 13:00 Joseph Le Potier *Strange duality for surfaces*

Break

15:00 - 15:30 Hideyasu Sumihiro *A splitting theorem of rank two vector bundles*

15:40 - 16:10 Wolfram Decker TBA

Break

16:45 - 17:15 Laurent Gruson *Deformation of instantons*

17:20 - 17:50 TBA

Jumping lines of Logarithmic bundles on $\mathbf{P^2}$

Jean Vallès
Université de Versailles, France
e-mail `valles@math.uvsq.fr`

In the paper *Arrangement of hyperplanes and vector bundles on* $\mathbf{P^n}$ the authors I.Dolgachev and M.Kapranov prove that the scheme of jumping lines of logarithmic bundles is uniquely defined by their finite set of superjumping lines. They show, for an appropriate first Chern class, that this scheme is a (so-called) monoïdal complex (Thm 5.2). In this talk I would like to show that on $\mathbf{P^2}$ the study of this monoïdal complex is related to the study of osculating hyperplane sections of $\mathbf{P^2}$ blown up along a finite subscheme.

On the Harder-Narasimhan Filtration of Principal Bundles

Vikram Mehta
Tata Institute for Fundamental Research, India
e-mail vikram@math.tifr.res.in

The stability and semistability of vector bundles has been studied for a long time. If a vector bundle is not semistable, then there is a canonical filtration, the Harder-Narasimhan filtration, which can be defined and which has certain rationality properties. In this talk we define and prove the rationality of the Harder-Narasimhan filtration for nonsemistable principal G-bundles, where G is any semisimple group. This is joint work with S.Subramanian

Holomorphic vector bundles

E. Ballico
Dept. of Mathematics, University of Trento, 38050 Povo (TN), Italy
e-mail `ballico@science.unitn.it`

Here I will present my recent work on holomorphic vector bundles on non-compact complex spaces (mainly on open subsets of Stein spaces and on quasi-affine algebraic varieties) and on infinite-dimensional complex manifolds.

The Moduli Problem for Instanton Bundles

Günther Trautmann
University of Kaiserslautern, Germany
e-mail `trm@mathematik.uni-kl.de`

Instanton bundles over the complex projective 3-space had been introduced in connection with Yang-Mills fields on the 4-sphere in the late 1970's. An n-instanton is defined to be a stable algebraic vector bundle of rank 2 over the projective 3-space with Chern classes $c_1 = 0$ and $c_2 = n > 0$. It is an open problem since the time when such bundles appeared, whether the moduli space $MI(n)$ of n-instantons is nonsingular or irreducible. Presently the answer is yes for $n \leq 5$. Together with a survey on instanton bundles a proof for this will be indicated stressing the recent efforts for the case $n = 5$.

Vector bundles and curves on hypersurfaces of $\mathbf{P}^4$

Luca Chiantini
Universitá di Siena, Italy
e-mail chiantini@unisi.it

Let X be a smooth hypersurface of degree d in $\mathbf{P}^4$. The study of vector bundles on X is relevant for our knowledge of subvarieties of X and their families. We focus mainly on rank 2 vector bundles E, which then correspond to subcanonical curves on X. When E is stable, indeed, its sections are almost in one-to-one correspondence with curves of this type and $\mathbf{P}H^0(E)$ can be viewed as a surrogate of a linear system of curves in codimension 2.

Horrock's celebrated splitting criterion says that bundles on $X = \mathbf{P}^3$ without intermediate cohomology, called arithmetically Cohen-Macaulay (ACM) bundles, split in a sum of line bundles. The criterion is known to fail as soon as $d = \deg(X) \geq 2$, but the exceptions must have Chern classes limited in a narrow numerical range. We review the bounds and the consequent classification of non-splitting ACM bundles on hypersurfaces of degree $d \leq 4$. For $d = 5$, the Calabi-Yau case, a complete classification in yet unavailable but, as a particular case of a conjecture of Tyurin, we prove that all stable ACM bundles are (infinitesimally) rigid in their Moduli space. Extensions of such results to general hypersurface of higher degree are discussed.

Finally we introduce a similar theory for the wider class of reflexive sheaves on X (which are associated to general curves) and discuss the study of these objects in connection with the theory of curves on some threefolds of general type.

Strange duality for surfaces

Joseph Le Potier
Institut de Mathématiques de Jussieu, France
e-mail: `jlp@math.jussieu.fr`

Let X be a complex algebraic smooth projective surface; assume for simplicity that X is simply connected. Denote by $K(X)$ the topological Grothendieck algebra of X; on $K(X)$ we have a natural quadratic form $c \mapsto \chi(c^2)$. Let M_c be the moduli space of semi-stable sheaves on X with fixed Grothendieck class in $K(X)$, respect to a generic polarization of X. If $c^* \in K(X)$ is orthogonal to c we can associate to c^* a line bundle $\mathcal{D}_{c^*}$ on M_c called determinant line bundle which generalizes the usual Donaldson determinant line bundle. With some other conditions we can define a natural bilinear form on $H^0(M_c, \mathcal{D}_{c^*}) \times H^0(M_{c^*}, \mathcal{D}_c)$. We explain some examples where this bilinear form gives a duality.

A splitting theorem of rank two vector bundles on projective spaces

Hideyasu Sumihiro
Hiroshima University, Japan
e-mail `sumihiro@math.sci.hiroshima-u.ac.jp`

As for splitting problems for rank two vector bundles on complex projective $n-$space $\mathbf{P}^n$,R.Hartshorne posed the following famous conjectures which are equivalent to each other.

S_n:*Every rank two vector bundle on* $\mathbf{P}^n$ $(n \geq 7)$ *splits into line bundles.*

C_n:*Every smooth closed subvariety of codimension 2 in* $\mathbf{P}^n$ $(n \geq 7)$ *is a complete intersection.*

Later, H.Grauert and M.Schneider tried to solve the following important problem. However there was a gap in their proof unfortunately.

GS:*Every unstable rank two vector bundle on* $\mathbf{P}^4$ *is a direct sum of line bundles.*

Though many mathematicians have attempted to solve the conjectures S_n, C_n and GS, one does not have obtained any complete answers yet.

We shall introduce determinantal varieties X associated to any very ample rank two vector bundle E on projective $n-$space $\mathbf{P}^n$ defined over an algebraically closed field of arbitrary characteristic which are smooth closed subvarieties of dimension m if $n = 2m$ in $\mathbf{P}^n$ (resp. $m + 1$ if $n = 2m + 1$) and investigate the following algebro-geometric properties of those determinantal varieties X : 1) Topology of X, 2) Divisors on X, 3) Comparison Theorems of Cohomologies, 4) Normal and Tangent bundles of X, 5) Hilbert Schemes of $\mathbf{P}^n$.

As a by-product, we can establish some splitting theorems for rank two vector bundles on $\mathbf{P}^n$ $(n \geq 4)$. The following splitting theorem is obtained by studying some geometric structures of the Hilbert scheme at determinantal varieties.

Theorem *Let* E *be a rank two vector bundle on* $\mathbf{P}^n$ $(n \geq 4)$ *defined over an algebraically closed field of arbitrary characterisic,* P *a 4- or 5- dimensional linear subspace of* $\mathbf{P}^n$ *and let* $\overline{E} = E|P$ *be the restriction of* E *to* P*, Then* E *is a direct sum of line bundles if and only if* $H^1(P, End(\overline{E})) = 0$.

TBA

Wolfram Decker
Universität des Saarlandes, Germany
e-mail `decker@math.uni-sb.de`

Deformation of Instantons

Laurent Gruson
Université de Versailles, France
e-mail gruson@math.uvsq.fr

I will discuss deformation of instantons into torsion free modules with dimension one singular locus. This locus acquires a theta-characteristic with additional restrictions.