Professor Carlo Sbordone
Dipartimento di Matematica "R. Caccioppoli"
Universitá di Napoli "Frederico II"
Via Cintia, 80126 Napoli
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Pittsburgh, April 12, 2002

Dear Professor Sbordone,

Please find enclosed the schedule and abstracts of the session entitled

Contemporary Developments in Partial Differential Equations and in the Calculus of Variations

to be held on Thursday, June 13, at the upcoming joint AMS-UMI meeting.

With very best wishes

Irene Fonseca and Paolo Marcellini (organizers)

Contemporary Developments in Partial Differential Equations and in the Calculus of Variations

Organizers: Irene Fonseca and Paolo Marcellini

Schedule

9:00 - 9:35 – D. Kinderlehrer

9:40 - 10:15 – G. Buttazzo

10:50 - 11:25 – G. Dolzmann

11:30 - 12:05 – G. Leoni

12:10 - 12:45 – G. Bellettini

15:00 - 15:35 – G. Dal Maso

15:40 - 16:15 – I. Fonseca

16:50 - 17:25 – C. Larsen

17:30 - 18:05 – P. Tilli

Contemporary Developments in Partial Differential Equations and in the Calculus of Variations

Organizers: Irene Fonseca and Paolo Marcellini

Abstracts

Some results on anisotropic geometric evolution problems

Giovanni Bellettini

We will deal with the problem of motion by mean curvature of a surface in presence of a possibly nonsmooth anisotropy. In particular, we will focus the attention on the evolution by crystalline mean curvature in three dimensions. The breaking and bending of a facet under this geometric flow will be discussed.

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Some results and questions in mass optimization problems

Giuseppe Buttazzo

We consider the optimization problem

$$\max\big\{\mathcal{E}(f,\Sigma,\mu)\ :\ \mu\in\mathcal{M}^+(m,\overline{\Omega})\big\}$$

where $\mathcal{E}(f,\Sigma,\mu)$ is the energy associated to μ:

$$\mathcal{E}(f,\Sigma,\mu)=\inf\big\{\frac{1}{2}\int|Du|^2\,d\mu-\langle f,u\rangle\ :\ u\in\mathcal{D}(\mathbf{R}^n),\ u=0\text{ on }\Sigma\big\}.$$

The datum f is a signed measure with finite total variation (and zero average if Σ is empty), while Ω is an open subset of $\mathbf{R}^n$, Σ is a closed subset of $\overline{\Omega}$, and $\mathcal{M}^+(m,\overline{\Omega})$ is the class of nonnegative measures on $\overline{\Omega}$ with mass m. We show that the optimization problem above admits a solution which is a measure, not in $L^1(\Omega)$ in general. This solution comes out by solving a mass transportation problem, for a suitable distance $d_{\Omega,\Sigma}$ and with the associated Monge-Kantorovich PDE equation of the form

$$\begin{cases} -\operatorname{div}\big(\mu D_\mu w\big) = f \quad \text{on } \mathbf{R}^n \setminus \Sigma \\ w \in \operatorname{Lip}_1(\Omega,\Sigma), \quad |D_\mu w| = 1 \ \mu\text{-a.e.}, \quad \mu(\Sigma) = 0 \end{cases}$$

where $\operatorname{Lip}_1(\Omega,\Sigma)$ is the class of all Lipschitz functions on Ω with constant 1, which vanish on Σ.

We also address some problems occurring in the optimization problems which arise when we consider the Dirichlet region Σ as an unknown.

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Neumann problems in domains with cracks and applications to fracture mechanics

Gianni Dal Maso

We consider solutions of nonlinear elliptic equations in domains of the form $\Omega \setminus K$, where Ω in a two dimensional smooth domain and K is a compact subset of $\overline{\Omega}$ with a finite number of connected components. The solutions are required to satisfy a homogeneous Neumann boundary condition on K and a nonhomogeneous Dirichlet condition on $\partial\Omega \setminus K$. The main result is the continuous dependence of the solution on K, with respect to the Hausdorff metric, provided that the number of connected components of K is uniformly bounded.

This stability result is used to give a rigorous mathematical formulation of a variational quasistatic model for the slow growth of brittle fractures, introduced by Francfort and Marigo. Starting from a discrete-time formulation, a more satisfactory continuous-time formulation is obtained, with full justification of the convergence arguments.

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Nematic elastomers

Georg Dolzmann

A modern trend in the calculus of variations which originates from applications in physics and materials science is to try to identify in a mathematically rigorous way limiting theories for complex systems. Often a lot of information about a given system can be obtained by analyzing a coarse-grained energy which describes the macroscopic behavior without resolving the finest length scale in the Integral representation and Γ-convergence of variational integrals with $p(x)$-growth

written in collaboration with Domenico Mucci, and submitted for publication in ESAIM:COCV, has entered the refereeing process. system. In this talk we discuss nematic elastomers as a model system for which the coarse-grained energy can be found analytically. We discuss the surprising predictions that result from this description.

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On Det versus det

Irene Fonseca

Sharp results for weak convergence of the determinant jacobian are given using carefully crafted isoperimetric inequalities. It is shown that if $u_n \in W^{1,N}(\Omega;\mathbb{R}^N)$, $u_n \rightharpoonup u$ in $W^{1,N-1}(\Omega;\mathbb{R}^N)$, where Ω is a bounded, open subset of $\mathbb{R}^N$, and if $\{\det \nabla u_n\}$ converges weakly-* in the sense of measures to a Radon measure μ, then $\frac{d\mu}{d\mathcal{L}^N} = \det \nabla u$ a.e. in Ω.

This is joint work with Giovanni Leoni and Jan Malý.

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Diffusion mediated transport and the brownian motor

David Kinderlehrer

We discuss a system of equations understood to describe the function of a typical protein motor, as described by Oster, Ermentrout, and Peskin and Adjari and Prost and their collaborators. An example of transport in an environment of diffusion, this mechanism governs many molecular level transduction processes and often falls under the rubric of the brownian motor or molecular rachet. What ingredients are essential in the function of these systems and why? We give an interpretation of the system, relating them to a variational principle involving Monge-Kantorovich mass transfer and the Wasserstein metric and describe our progress toward understanding the transport properties. This is joint work with Michel Chipot and Michal Kowalczyk.

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Transfer of jump sets for convergent sequences in SBV

Christopher Larsen

In proving existence for mathematical models of brittle fracture growth, the following problem arises:

Suppose $u_n \in SBV(\Omega)$ minimizes

$$E(v) := \int_\Omega |\nabla v|^2 dx + \mathcal{H}^{N-1}(S_v \backslash S_{u_n})$$

subject to $v = u_n$ on $\partial\Omega$, where $\Omega \subset I\!R^N$. If $u_n \to u$ (in the sense of SBV compactness), does the corresponding minimality hold for u?

A partial answer has been provided by Dal Maso and Toader, who proved this minimality for $N = 2$ assuming that the number of components of S_{u_n} is bounded. For the general result, our proof relies on a method of transferring jump sets. In particular, for any $\phi \in SBV$, we construct a sequence $\phi_n \to \phi$ such that

$$\lim_{n\to\infty} \mathcal{H}^{N-1}(S_{\phi_n} \backslash S_{u_n}) \leq \mathcal{H}^{N-1}(S_\phi \backslash S_u).$$

In this talk, I will explain the proof and its connection to the fracture problem. This is part of joint work with G. Francfort.

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New lower semicontinuity results in the Calculus of Variations

Giovanni Leoni

We discuss lower semicontinuity properties for integral functionals which are non-coercive or satisfy non-standard growth conditions.

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A variational approach to the Hele-Shaw flow with injection

Paolo Tilli

We describe a variational approach to the Hele-Shaw flow with injection.

After giving a suitable weak formulation of the evolution equation, we construct e sequence of approximating solutions to the Hele-Shaw flow, depending on a discretization step δt, by iteratively solving a variational problem.

As the time step tends to zero, the approximating solutions converge increasingly (with respect to set inclusion, at fixed time) to a weak solution of the problem. When the latter is not unique, the solution thus obtained is characterized by a minimality property, with respect to set inclusion, at fixed time.

We also prove several monotonicity results of the weak solutions, with respect to both the initial initial set and the forcing term. In particular, the monotonicity property with respect to the initial set implies that the interface has finite perimeter at every time, provided the starting set has finite perimeter.

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