Kinetic Models of Granular Flows

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We review recent results on one-dimensional kinetic models of the Boltzmann equation with dissipative collisions and variable coefficient of restitution. In particular, we investigate the behavior of the Boltzmann equation in the so-called quasi elastic limit for a wide range of the rate function. By this limit procedure we obtain a class of nonlinear equations classified as nonlinear friction equations. We analyze the long-time asymptotics of these nonlinear friction equations, which split naturally into two classes, which are characterized by the behavior in time of their similarity solutions (homogeneous cooling state). If the similarity solution relaxes towards equilibrium in infinite time, we show convergence in the Vasershtein metric of any solution with the same mass and mean velocity towards the similarity solution by computing explicit relaxation rates. Finally we introduce numerical schemes for these dissipative models. We study the numerical passage of the Boltzmann equation with singular kernel to nonlinear friction equations in the quasi elastic limit. To this aim we introduce a Fourier spectral method for the Boltzmann equation and show that the kernel modes that define the spectral method have the correct quasi elastic limit providing a consistent spectral method for the limiting nonlinear friction equation.