# Gap functions and Lyapounov functions

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**Abstract:**

Equilibrium problems play a central role in the study of complex and competitive systems. There are many classical examples both in engineering science (traffic network equilibrium problems, for example), and in economics (oligopolistic market equilibrium, for example). Many variational formulations of these problems have been presented in these years. So, variational inequalities are very useful tools for the study of equilibrium solutions and their stability. More recently two dynamical models of equilibrium problems based on projection operators were proposed. The first type is known as locally projected dynamical system, the second one is designated as globally projected dynamical system. The equilibrium points of these systems are the solutions to the associated variational inequality (VI) problem. A very popular approach for finding solution of these VI and for studying its stability consists in introducing the so-called "gap-functions", while stability analysis of an equilibrium point of dynamical systems can be made by means of Lyapunov functions. We here show strict relationships between gap functions and Lyapunov functions

**Key words:** variational inequalities, gap functions, projected dynamical systems, equilibrium solutions, stability analysis, Lyapunov functions.

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