

Mathematical aspects of statistical mechanics

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PhD Course in Mathematics @ Unipi
2026-2027

The class is open to master's students (for master's students the course will grant 6 CFU).

Course description

The goal of this course is to provide an introduction to the rigorous mathematical formulation of statistical mechanics. We will develop the fundamental tools needed to study the collective behavior of many-body systems, including the thermodynamic limit, equilibrium states, phase transitions, and spontaneous symmetry breaking. Paradigmatic models, such as classical lattice spin systems (e.g., the Ising model) and the quantum Bose gas, will be used to illustrate these concepts.

A brief list of topics includes:

- Introduction to the Ising model and the quantum Bose gas
- Existence of the thermodynamic limit
- Correlation inequalities
- Existence of phase transitions in the low-temperature regime, Peierls argument
- Algebraic characterization of equilibrium states (Gibbs measures, ergodic states, KMS condition)

Class requirements

No previous knowledge of statistical mechanics will be assumed. For most of the lectures solid background in analysis and probability will be sufficient. Knowledge of basic quantum mechanics will be helpful (but not required) when discussing quantum models. Specific results will be recalled when needed.

Course Schedule

30 hours, from the second week of November, 2026 until January, 2027.

Exam information

Oral exam.

References

The course does not follow a particular textbook. The topics covered in the course (along with much else) are almost all covered in the textbooks below.

Main references

- [1] D. Ruelle, *Statistical Mechanics: Rigorous Results*, 1969.
- [2] S. Friedli and Y. Velenik, *Statistical Mechanics of Lattice Systems: A Concrete Mathematical Introduction*, 2017.

Other references

- [3] G. Gallavotti, *Statistical Mechanics: A Short Treatise*, 1999.
- [4] J. Glimm and A. Jaffe, *Quantum Physics: A Functional Integral Point of View*, 2nd ed., 1987.
- [5] B. Simon, *The Statistical Mechanics of Lattice Gases, Volume I*, 1993.
- [6] A. Ya. Khinchin, *Mathematical Foundations of Statistical Mechanics*, 1949.

Contacts

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