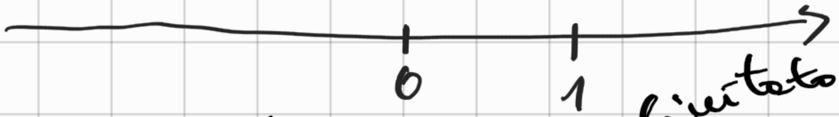
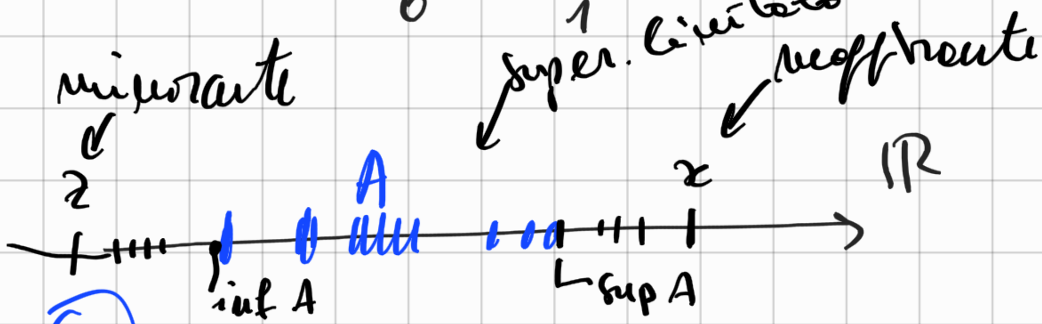
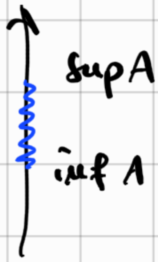
ANALISI MATEMATICA B
LEZIONE 15 — 22.10.2021







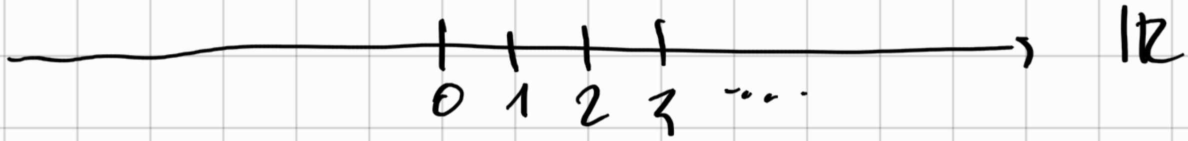
$A \subseteq \mathbb{R}$ $\qquad x \in \mathbb{R}$ $\qquad x \geq A$

$\forall a \in A : x \geq a$

$\sup A = \min \{\text{maggioranti di } A\}$.

$\inf A = \max \{\text{minoranti di } A\}$.

Proprietà Archimedea di $\mathbb{R}$

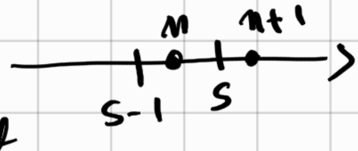


$\nexists\, x \in \mathbb{R} : \forall n \in \mathbb{N} : x \geq n$

dim per assurdo $\exists\, x \in \mathbb{R} : \forall n \in \mathbb{N} : x \geq n$

$\mathbb{N}$ è superiormente limitato

$s = \sup \mathbb{N} \qquad s \in \mathbb{R}$.



$s-1$ non è un maggiorante

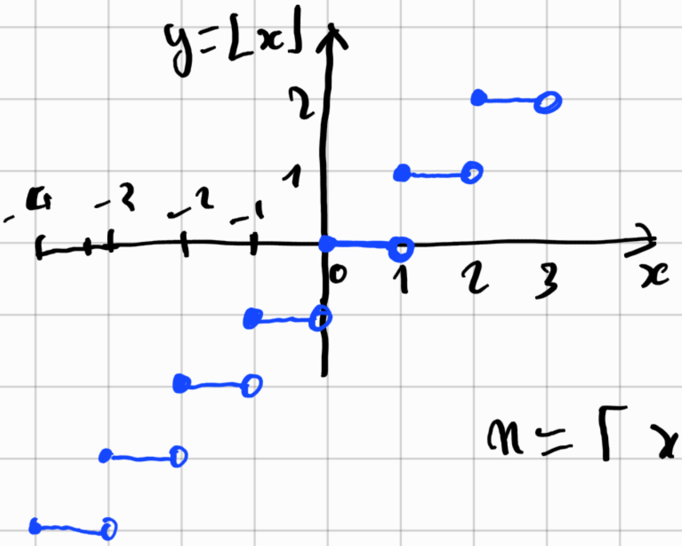
$\exists\, n \in \mathbb{N} : s-1 < n \Rightarrow s < n+1$ assurdo $\square$

Corollario Se $\varepsilon > 0$, $\varepsilon \in \mathbb{R}$, $\exists\, n \in \mathbb{N}$ t.c. $\frac{1}{n} < \varepsilon$.

Def Parte intera $x \in \mathbb{R}$ $\cancel{(x>0)}$

$\exists\, n \in \cancel{\mathbb{N}}\ \mathbb{Z}$ $\quad n \leq x < n+1$ $\quad n = \lfloor x \rfloor$

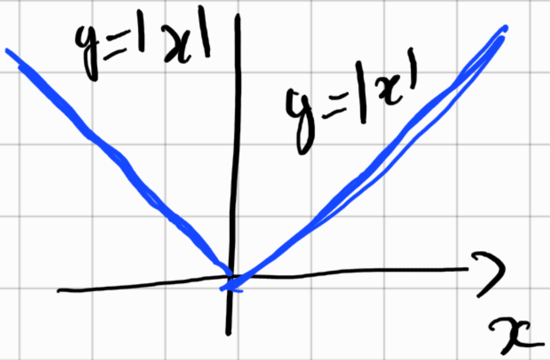
$x = n + (x-n)$



$\lfloor -3.14 \rfloor = -4$

$n = \lceil x \rceil$ se $n-1 < x \leq n$

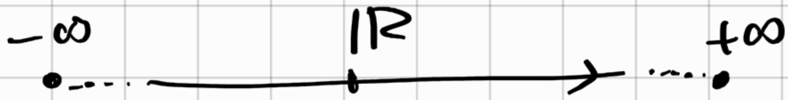
$$\lfloor x \rfloor \leq x \leq \lceil x \rceil$$



$$|x| = \begin{cases} x & \text{se } x \geq 0 \\ -x & \text{se } x < 0 \end{cases}$$

Reali estesi $\overline{\mathbb{R}}$

$$\overline{\mathbb{R}} = \mathbb{R} \cup \{+\infty, -\infty\}$$



ordinamento su $\overline{\mathbb{R}}$ è lo stesso di $\mathbb{R}$ in più:

$$+\infty > x \quad \forall x \in \mathbb{R}$$
$$-\infty < x \quad \forall x \in \mathbb{R}$$
$$-\infty < +\infty .$$

Se $A \subseteq \mathbb{R} \subseteq \overline{\mathbb{R}}$ $\quad A \leq +\infty$

$s = \sup A$ esiste sempre $s \in \overline{\mathbb{R}}$

se A è sup. limitato $s < +\infty$

se A non è sup. limitato $s = +\infty$

se $A = \emptyset$ l'insieme dei maggioranti è $\mathbb{R}$ o $\overline{\mathbb{R}}$

$\sup \emptyset = -\infty$

Se A è inf. limitato e non vuoto $\inf A \in \mathbb{R}$

Se A non è inf. limitato $\inf A = -\infty$

Se $A = \emptyset$ $\quad \inf A = +\infty$

Possiamo definire + e · su $\overline{\mathbb{R}}$?

$(+\infty) + x = +\infty$ se $x \neq -\infty$

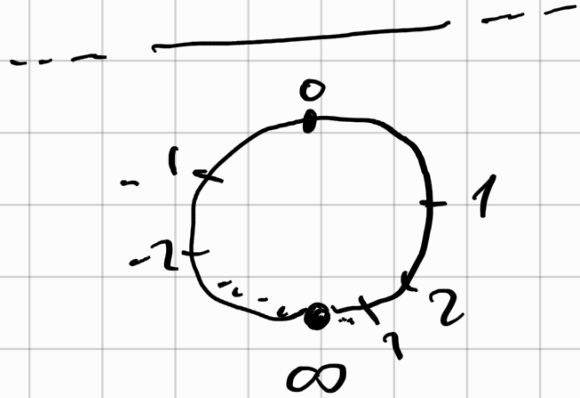
$(-\infty) + x = -\infty$ se $x \neq +\infty$

$$x \cdot (+\infty) = \begin{cases} +\infty & \text{se } x > 0 \\ \text{non def.} & \text{se } x = 0 \\ -\infty & \text{se } x < 0 \end{cases}$$

$-(+\infty) = -\infty$ $\qquad$ $-(-\infty) = +\infty$

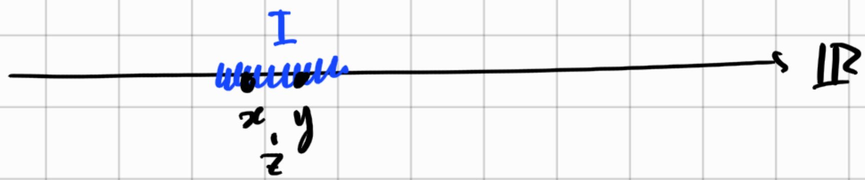
$\frac{1}{+\infty} = 0 = \frac{1}{-\infty}$ $\qquad$ $\frac{1}{0}$ non si definisce.

Cos'è ∞ ? $\mathbb{R}$



NON HO ORDINAMENTO.

Intervalli



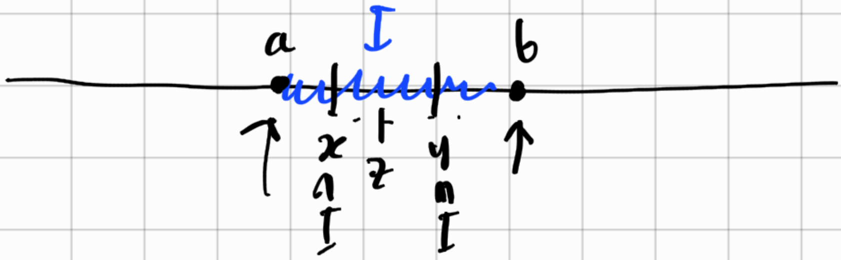
$I \subseteq \mathbb{R}$ si dice essere un intervallo se vale la proprietà dei valori intermedi:

$$x, y \in I \Rightarrow \forall z \in \mathbb{R} : x \leq z \leq y \Rightarrow z \in I.$$

Proposizione Se I è un intervallo
posto $a = \inf I$, $b = \sup I$

$$\forall z \in \mathbb{R} : \quad a < z < b \Rightarrow z \in I.$$
$$z > b \Rightarrow z \notin I$$
$$z < a \Rightarrow z \notin I$$

dim

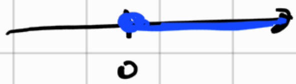


□

Classificazione: I intervallo, $a = \inf I$, $b = \sup I$

$$I = \begin{cases} \{x \in \mathbb{R} : a < x < b\} =: (a,b) =]a,b[\\ \{x \in \mathbb{R} : a \le x \le b\} =: [a,b] \\ \{x \in \mathbb{R} : a < x \le b\} =: (a,b] =]a,b] \\ \{x \in \mathbb{R} : a \le x < b\} =: [a,b) = [a,b[\end{cases}$$

ES $[0,+\infty) = \{x \in \mathbb{R} : 0 \le x\}$



ES $(-\infty,+\infty) = \mathbb{R}$.

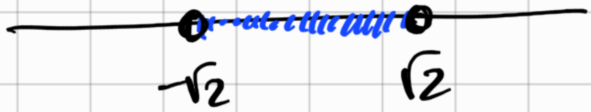
ES $[0,1] = \{x \in \mathbb{R} : 0 \le x \le 1\}$

...

$\emptyset$ è un intervallo $\quad$ " $\emptyset = (+\infty,-\infty)$
$= (1,-1)$ "

$[a,a] = \{a\}$ è un intervallo.

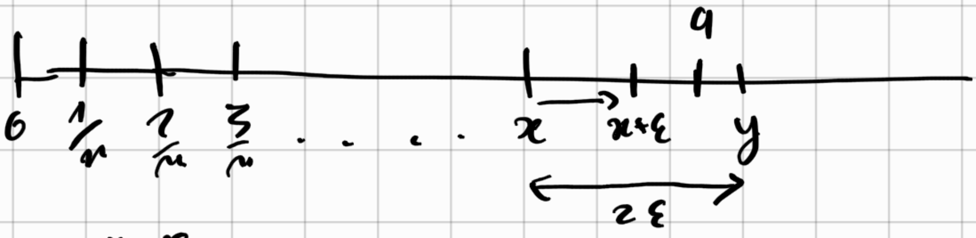
Es in $\mathbb{Q}$ gli intervalli potrebbero non avere estremi. Es: $I = \{q \in \mathbb{Q} : q^2 < 2\}$



Proposizione $\mathbb{Q}$ è denso in $\mathbb{R}$.

Se $x < y$, $x, y \in \mathbb{R}$ $\exists q \in \mathbb{Q} : x < q < y$

dim



$\varepsilon = \frac{y-x}{2}$ $\quad \exists n \in \mathbb{N} : \quad \frac{1}{n} < \varepsilon$

$$q = \frac{\lfloor (x+\varepsilon) \cdot n \rfloor}{n}$$

$q \geq x + \varepsilon > x$

$q < x + \varepsilon + \frac{1}{n} < x + 2\varepsilon = y$

$\square$

Es $\quad x = 3.1415 \qquad \frac{p}{37}$ più vicina a x?

$$p = \begin{cases} \frac{\lfloor x \cdot 37 \rfloor}{37} \\ \frac{\lceil x \cdot 37 \rceil}{37} \end{cases}$$

Esercizio Se $x < y$, $x, y \in \mathbb{Q}$ esiste $z \in \mathbb{R} \setminus \mathbb{Q}$ tale che $x < z < y$.

Funzioni quadratiche

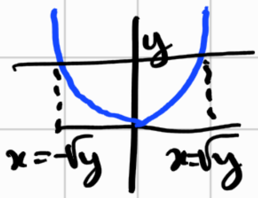
$$f(x) = \underbrace{a\cdot x^2 + b\cdot x + c} \qquad a \neq 0$$

Per invertire f per ogni $y \in \mathbb{R}$ dovrei trovare

$x \in \mathbb{R}$: $f(x) = y$. $\qquad a\cdot x^2 + b\cdot x + c - y = 0$

Basta trovare gli zeri: $\{x \in \mathbb{R} : f(x) = 0\}$

So invertire $g(x) = x^2$



$$x^2 = y \qquad \begin{cases} x = \pm\sqrt{y} & \text{se } y > 0 \\ x = 0 & \text{se } y = 0 \\ \nexists x & \text{se } y < 0 \end{cases}$$

$$(x+d)^2 = x^2 + 2dx + d^2 \qquad d = \frac{b}{2a}$$

$$ax^2 + bx + c = a\left[x^2 + \underbrace{\frac{b}{a}x} + \frac{c}{a}\right]$$

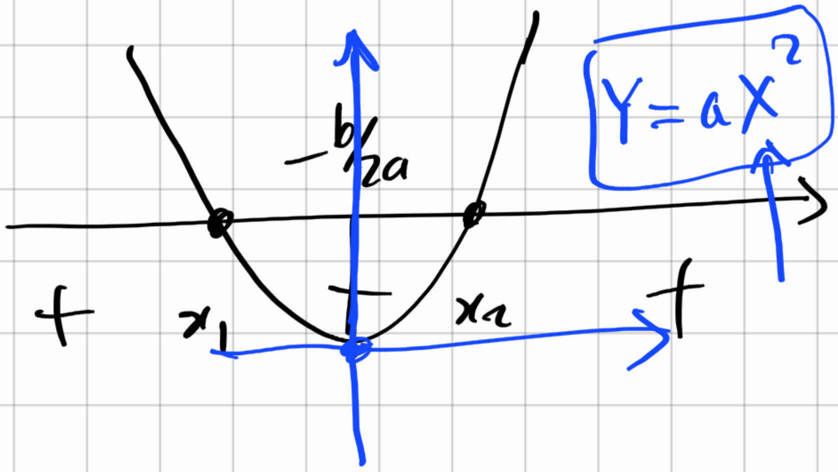
$$= a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a}\right] \overset{?}{\gtrless} 0$$

$$\left(x + \frac{b}{2a}\right)^2 \gtrless \frac{b^2}{4a^2} - \frac{c}{a} = \frac{b^2 - 4ac}{4a^2} = \frac{\Delta}{4a^2}$$

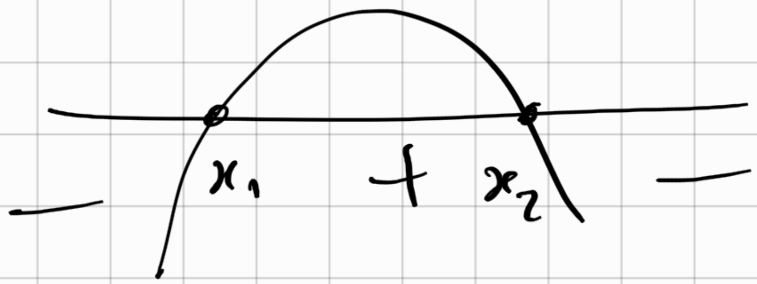
dipende dal segno di Δ $\qquad \Delta = b^2 - 4ac$

$$\begin{cases} \text{Se } \Delta > 0 & x + \frac{b}{2a} = \pm\sqrt{\frac{\Delta}{4a^2}} = \frac{\pm\sqrt{\Delta}}{2a} \\ & x_{12} = \frac{-b \pm \sqrt{\Delta}}{2a} \\ \text{Se } \Delta = 0 & x + \frac{b}{2a} = 0 \qquad x = -\frac{b}{2a} \end{cases}$$

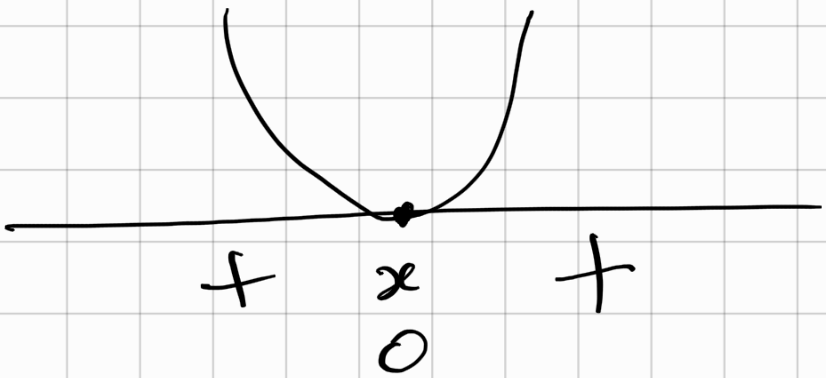
se $\Delta < 0$ non ho soluzioni.



se $\Delta > 0$
se $a > 0$

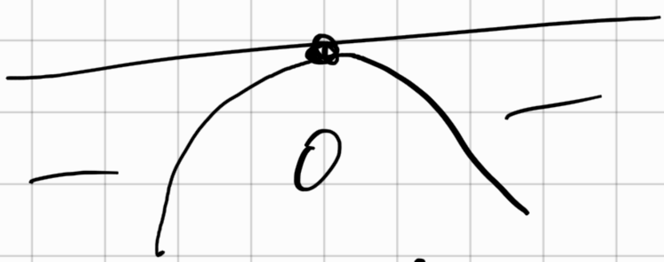


se $\Delta > 0$
se $a < 0$



se $\Delta = 0$
$a > 0$

se $\Delta = 0$
$a < 0$



se $\Delta < 0$
$a > 0$

se $\Delta < 0$
$a < 0$