ANALISI MATEMATICA B
LEZIONE 36 15.12.2021

ES

$$\begin{cases} a_0 = \alpha \\ a_{n+1} = \dfrac{3}{3-a_n} - 1 \end{cases}$$

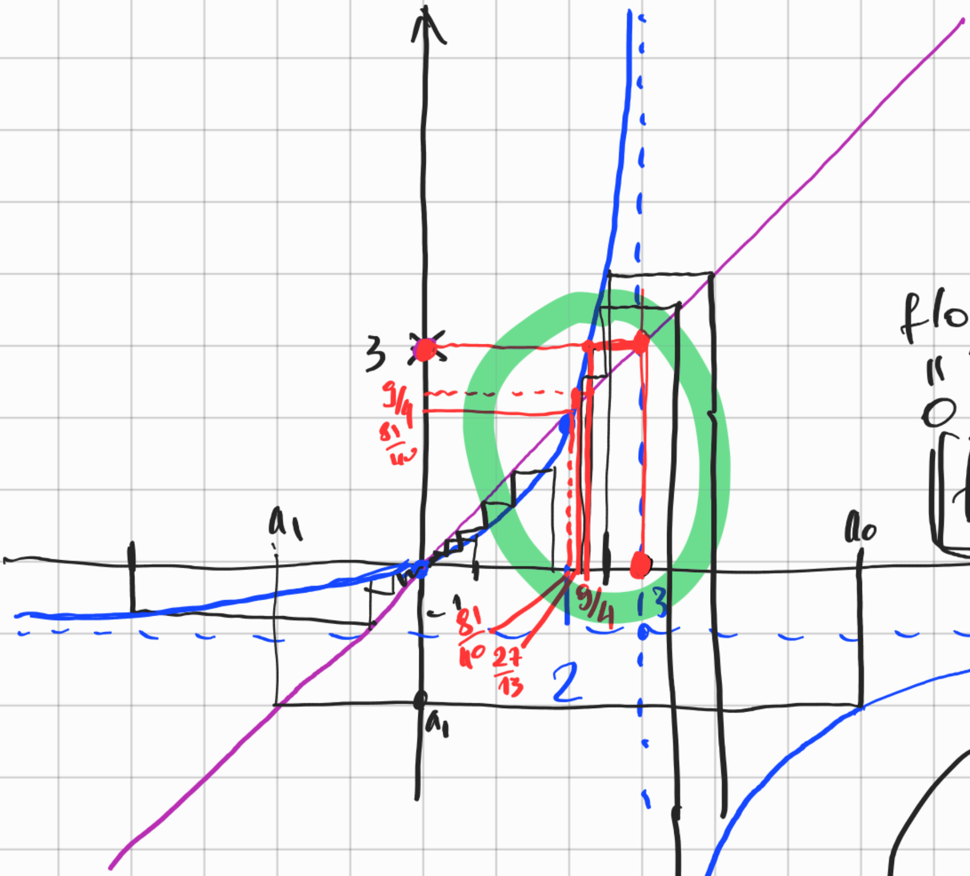
(1) $\alpha \in (-\infty, 2] \cup (3, +\infty)$ ✓

(2) $\alpha \in \left[\frac{81}{40}, 3\right]$ ←

$a_{n+1} = f(a_n)$

$$f(x) = \frac{3}{3-x} - 1$$

$$\frac{3}{3-x} - 1 > x$$



$0 \le x \le 2$ stett. $f(x)$ è crescente su $(-\infty, 3)$

$$f(0) \le f(x) \le f(2)$$
$$\;\;\shortparallel \qquad\qquad \shortparallel$$
$$\;\;0 \qquad\qquad\;\; 2$$

$\boxed{f(x) \le x}$ se $x \in [0, 2]$

$[0, 2]$ è invariante

se $\alpha \in [0, 2] \Rightarrow a_n \in [0, 2]$ $\forall n$.

$a_{n+1} = f(a_n) \le a_n$ $\quad$ a_n decrescente $\quad$ $a_n \to \ell \in [0, 2]$

$$a_{n+1} = f(a_n)$$
$$\downarrow \qquad \downarrow$$
$$\ell = f(\ell)$$

ℓ è un punto fisso

$\ell = 0$ o $\ell = 2$

Se $\boxed{\alpha < 2}$ $\quad a_n \le a_0 = \alpha < 2$

$\ell \le \alpha \qquad \ell \ne 2$

Se $\alpha = 2$ $\quad a_n = 2 \qquad \ell = 2$.

Se $\alpha < 0$, $\quad (-\infty, 0]$ è invariante

$f(x) > x$ su $(-\infty, 0)$

a_n è crescente < 0 $\qquad a_n \to \overset{\le 0}{\ell}$ finito

ℓ pto fisso

$\ell = 0$.

Se $\alpha > 3$ $\qquad a_0 = \alpha \qquad a_1 = f(\alpha) < -1$

si torna al caso precedente

$x > 3 \Rightarrow f(x) < -1$

f manda $(3, +\infty)$ in $(-\infty, 1)$

Risolvo: $f(x)=3$

Se $\alpha = \frac{9}{4} > \frac{81}{40}$

la successione non è ben definita

$$\frac{3}{3-x} - 1 = 3$$

$$\frac{3}{3-x} = 4$$

$$4(3-x) = 3$$

$$12 - 4x = 3$$

$$4x = 9$$

$$x = \frac{9}{4}$$

Risolvo $f(x) = \frac{9}{4}$

$$\frac{3}{3-x} - 1 = \frac{9}{4}$$

$$\frac{3}{3-x} = \frac{13}{4}$$

$$12 = 13(3-x)$$

$$12 = 39 - 13x$$

$$x = \underset{2+\frac{1}{13}}{\frac{27}{13}} > \frac{81}{40} = 2 + \frac{1}{40}$$

$$2 + \frac{1}{n}$$

Risolvo $f(x) = \frac{27}{13}$

$$\frac{3}{3-x} - 1 = \frac{27}{13}$$

$$\frac{3}{3-x} = \frac{40}{13}$$

$$39 = 40(3-x)$$

$$39 = 120 - 40x$$

$$x = \frac{81}{40}$$

$$\frac{3}{3-x} - 1 = y$$

$$\frac{3}{3-x} = y + 1$$

$$3 = (3-x)(y+1)$$

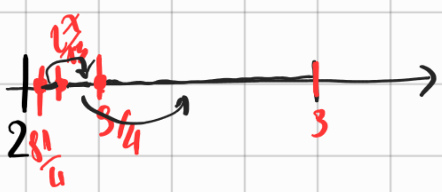
$$\cancel{3} = 3y + \cancel{3} - x(y+1)$$

$$x = \frac{3 \cdot y}{y+1}$$

$$f^{-1}(y) = \frac{3y}{y+1} = 3 - \frac{3}{y+1}$$

$\frac{81}{40}$, $\frac{27}{13}$, $\frac{9}{4}$, 3 sono "cattivi"

(se α è uno di questi



la succ. a_n non è definita per tutti gli n.

$$\left(\frac{81}{40}, \frac{27}{13}\right) \xrightarrow{f} \left(\frac{27}{13}, \frac{9}{4}\right) \xrightarrow{f} \left(\frac{9}{4}, 3\right) \xrightarrow{f} (3, +\infty)$$

$$\downarrow$$

$$(-\infty, -1)$$

$\underbrace{}_{\text{INVARIANTE}}$

$a_n \to 0$

$$\frac{81}{40} < x < \frac{27}{13}$$

$$f\left(\frac{81}{40}\right) < f(x) < f\left(\frac{27}{13}\right)$$

$$f\left(\frac{81}{40}\right) = \frac{27}{13}, \qquad f\left(\frac{27}{13}\right) = \frac{9}{4}$$

$$\begin{cases} b_0 = 3 \\ b_{n+1} = \dfrac{3b_n}{b_n+1} \end{cases}$$

$\leftarrow$ è la succ. dei punti cattivi.

$$\boxed{b_n \overset{?}{=} 2 \cdot \frac{3^{n+1}}{3^{n+1}-1}} \leftarrow \quad b_{n+1} = 2 \cdot \frac{3^{n+2}}{3^{n+2}-1}$$

$$b_{n+1} \overset{!}{=} \frac{3b_n}{b_n+1} = 3 - \frac{3}{b_n+1} = 3 - \frac{3}{2\cdot\frac{3^{n+1}}{3^{n+1}-1}+1}$$

ipotesi induttiva

$$= 3 - \frac{3}{\frac{2\cdot 3^{n+1} + 3^{n+1} - 1}{3^{n+1}-1}} = 3 - \frac{3}{\frac{3^{n+2}-1}{3^{n+1}-1}}$$

$$= 3 - \frac{3(3^{n+1}-1)}{3^{n+2}-1} = \frac{3\cdot 3^{n+2} \cancel{-3} - 3^{n+2} \cancel{+3}}{3^{n+2}-1}$$

$$= \frac{2\cdot 3^{n+2}}{3^{n+1}-1}$$ ☺ ok!!

$$\sum_{n=0}^{+\infty}\left(x^n+\frac{1}{x^n}\right) \qquad x\neq 0$$

$x<-1$ Σ indet

$$\left|x^n+\frac{1}{x^n}\right| \to \begin{cases} +\infty & |x|>1 \\ 2 & x=1 \\ +\infty & |x|<1 \\ 2 & x=-1 \end{cases}$$

Σ diverge

$|x|<1$ Σ diverge

$x=-1$ Σ indet

$$\sum_{n=0}^{+\infty}\frac{x^n}{1+x^n}$$

$$\frac{x^n}{1+x^n} = \frac{1}{\frac{1}{x^n}+1}$$

$|x|>1$ Σ diverge

$|x|<1$ Σ converge

$1+x^n \sim 1$

ES

$$\boxed{\lim_{n\to+\infty}\sqrt[n]{a^n+b^n}}$$