ANALISI MATEMATICA B
LEZIONE 79 – 13.4.2022

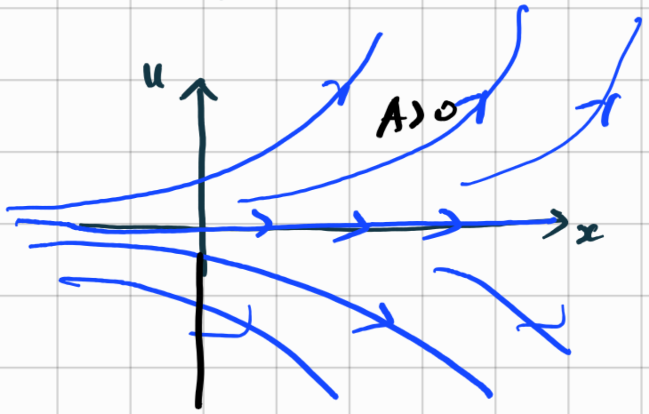
Modelli (biologici)

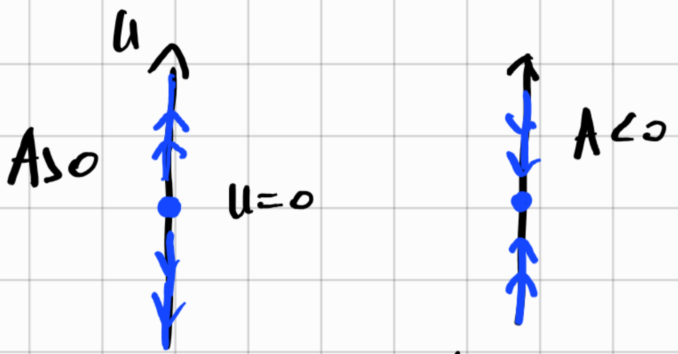
Crescita esponenziale   Malthus

$u(x)$ = # di individui al giorno x

$u'(x) = A u(x)$ A = tasso di crescita

$$u(x) = C \cdot e^{Ax} = \pm e^{A(x-x_0)}$$

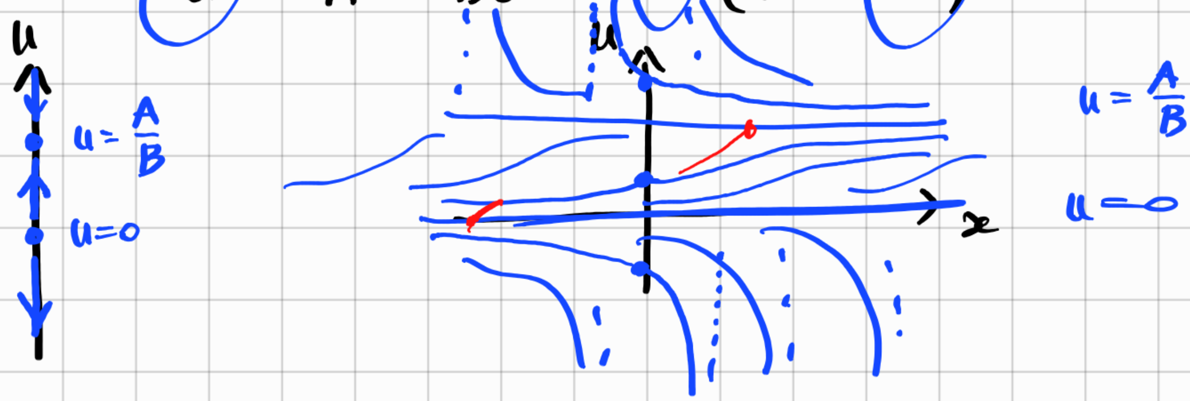




Equazione logistica

$$u'(x) = A u(x) - B u^2(x)$$

$$u' = Au - Bu^2 = u(A - Bu)$$



$u(0) \in \left(0, \frac{A}{B}\right) \Rightarrow$ u è definita su tutto $\mathbb{R}$
u è strettamente crescente

$$\lim_{x\to+\infty} u(x) = \ell \in \left[0, \frac{A}{B}\right]$$

Dimostriamo che $\ell = \frac{A}{B}$. per $x \to +\infty$ $u \to \ell$

$$u' = u \cdot (A - Bu)$$
$$\downarrow$$
$$u' \to \ell \cdot (A - B\ell)$$

Ma u ha un asintoto orizz. $\Rightarrow \underbrace{\ell \cdot (A - B\ell)} = 0$

$A=1 \quad B=1$

$u' = u - u^2$ $\qquad \int \frac{u'}{u-u^2} = \int 1 \ \ldots$

[....]

$$u(x) = 1 \pm \frac{1}{c \cdot e^x} \quad [?).$$

Equazione del pendolo:

$u'' = -\sin u$
$v = u'$

$\underline{u} = \begin{pmatrix} u \\ v \end{pmatrix}$

$$\begin{cases} v' = -\sin u \\ u' = v \end{cases}$$

$v' = -\sin u$
$v' v = -\sin u \cdot u'$ ← integrando in dx
$u'dx = du$
$\frac{1}{2} v^2 = \cos u + c$

Energia potenziale

$$E(u,v) = \boxed{\frac{1}{2}v^2 - \cos u = c}$$

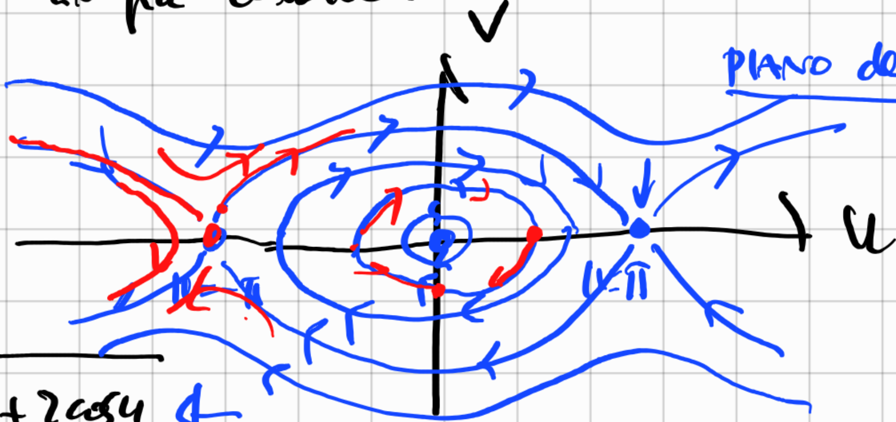
Energia cinetica

Sol. stazionarie

$v = 0$

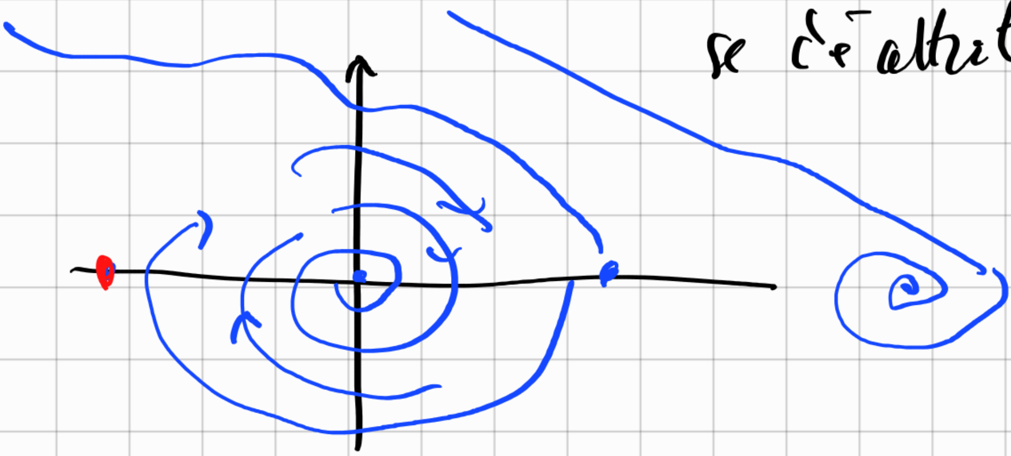
$u = k\pi$

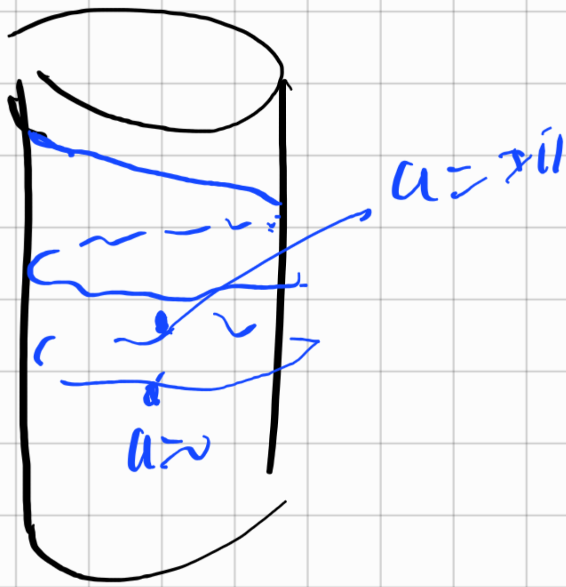
PIANO delle FASI



$$v = \pm\sqrt{2c + 2\cos u}$$

se c'è attrito





• Modello preda/predatore di Volterra-Lotka.

$u = u(x)$ quantità di prede

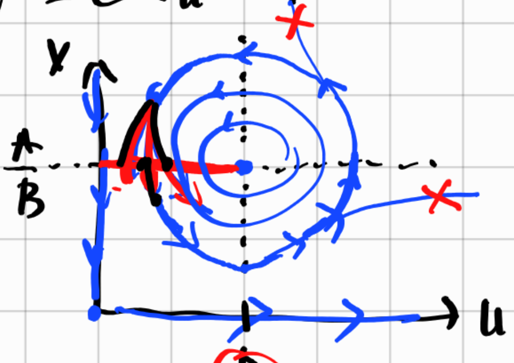
$v = v(x)$ " " predatori

$$\begin{cases} u' = Au - B\cdot uv \\ v' = C\cdot vu - Dv \end{cases}$$

$v = \frac{A}{B}$

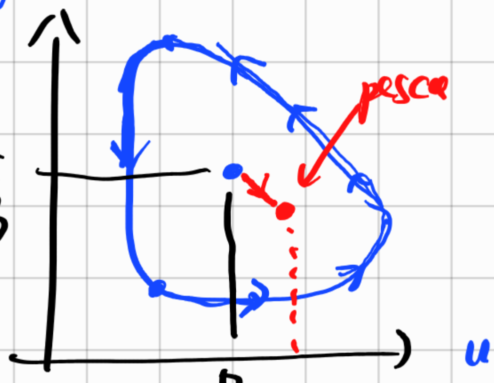
$$\begin{cases} u' = u\cdot(A - Bv) \\ v' = v\cdot(Cu - D) \end{cases}$$

$u = \frac{D}{C}$



$u' > 0$

$v' > 0$



$$\begin{cases} u' = u\cdot(A - Bv) \\ v' = v\cdot(Cu - D) \end{cases}$$

$$\frac{v'}{u'} = \frac{v\cdot(Cu - D)}{u\cdot(A - Bv)}$$

$$\int \left[\frac{v'}{v}(A - Bv) = \frac{u'}{u}(Cu - D) \right] dx$$

$u = u(x)$

$du = u'(x)\, dx$

$$\int \frac{1}{v}(A - Bv)\, dv = \int \frac{Cu - D}{u}\, du$$

$$\int \left[\frac{A}{v} - B \right] dv = \int \left(C - \frac{D}{u} \right) du$$

$$A \ln v - Bv = Cu - D \ln u + K$$

$$H(u,v) = \boxed{A\ln v + D\ln u - Bv - Cu}$$

$$H(u(x), v(x)) = K$$

$$f(u) = H\left(u, \frac{A}{B}\right) = A\ln\frac{A}{B} + D\ln u - \cancel{B}\cdot\frac{A}{\cancel{B}} - Cu$$

$$f'(u) = D\cdot\frac{1}{u} - C$$

u		D/C	
$f'(u)$	$+$	0	$-$
$f(u)$	$\nearrow$		$\searrow$

$$\frac{D}{u} = C$$

$$u = \frac{D}{C}$$

$$\begin{cases} u' = Au - Buv - Eu = (A-E)u - Buv \\ v' = Cuv - Dv - Ev = Cuv - (D+E)v \end{cases}$$