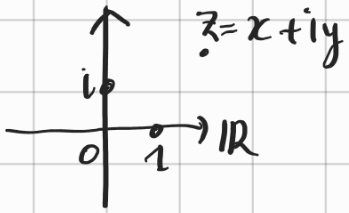
ANALISI MATEMATICA B
LEZIONE 16 – 28.10.2022



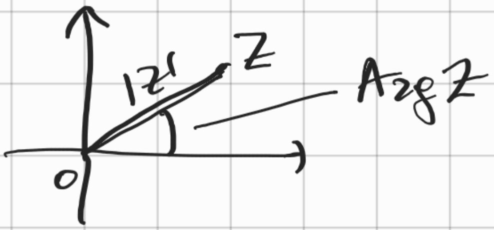
$i^2=-1$

$z=x+iy$

$\bar z = x-iy$

$|z| = \sqrt{x^2+y^2} = \sqrt{z\cdot\bar z}$

$|z\cdot w| = |z|\cdot|w|$



$\mathrm{Arg}\, z\cdot w = \mathrm{Arg}\, z + \mathrm{Arg}\, w.$

$\boxed{z^n = c}$

Esercizio Risolvere $z^3=i$

Metodo brutale: $z=x+iy$

$$z^3 = (x+iy)^3 = x^3+3x^2iy+3x(iy)^2+(iy)^3$$
$$= x^3+3x^2y\,i-3xy^2-iy^3$$
$$= \underbrace{x^3-3xy^2} + i\underbrace{(3x^2y-y^3)} \overset{!}{=} i$$

$$\begin{cases} x^3-3xy^2=0 \\ 3x^2y-y^3=1 \end{cases}$$

I $\begin{cases} x=0 \\ y^3=-1 \end{cases}$ $\begin{cases} x=0 \\ y=-1 \end{cases}$

$z=-i$

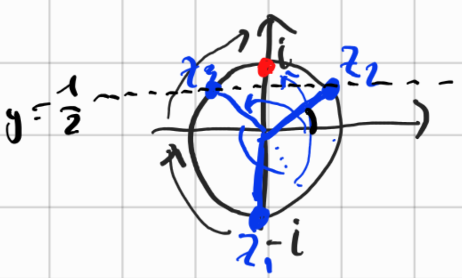
$\begin{cases} x^2-3y^2=0 \\ \% \end{cases}$

$\begin{cases} x^2=3y^2 \\ 9y^3-y^3=1 \end{cases}$

$\begin{cases} \% \\ (2y)^3=1 \end{cases}$

$\begin{cases} x^2=\frac{3}{4} \\ y=\frac{1}{2} \end{cases}$

II $\begin{cases} x=\frac{\sqrt3}{2} \\ y=\frac12 \end{cases}$ III $\begin{cases} x=-\frac{\sqrt3}{2} \\ y=\frac12 \end{cases}$



$z^3=i$

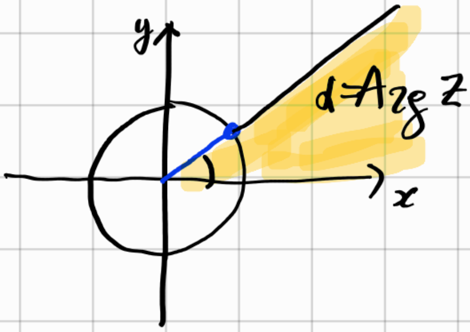
$z^3-i=0$

$z^3-(-i)^3=0$

$(z+i)(z^2-iz-1)=0$

$a^3-b^3=(a-b)(a^2+ab+b^2)$

FUNZIONI TRIGONOMETRICHE   sin cos



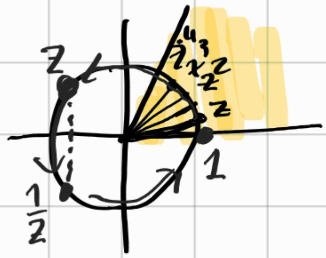
$z = x+iy$ $|z|=1$ $x^2+y^2=1$

$\cos\alpha = x$ $\cos^2\alpha + \sin^2\alpha = 1.$

$\sin\alpha = y$ $|\cos\alpha| \le 1$

$|\sin\alpha| \le 1$

COME MISURO UN ANGOLO?



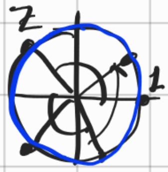
$|z|=1$

$|z^2|=1$

$z^{360}=1$

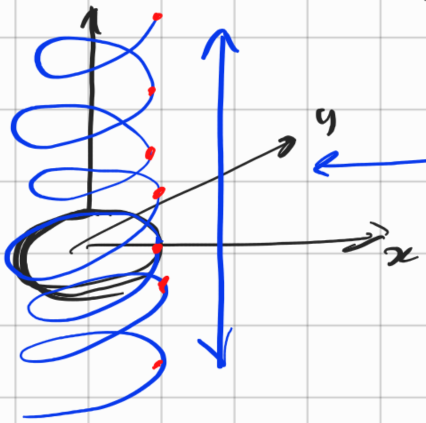
$\mathcal{U} = \{z\in\mathbb{C} : |z|=1\}$ $z,w\in\mathcal{U} \Rightarrow z\cdot w \in \mathcal{U}$

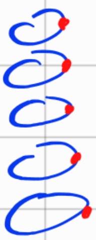
$\mathcal{U}$ con $\cdot$ è un gruppo (è un "sottogruppo" di $\mathbb{C}$)



$$\frac{1}{z} = \frac{\bar z}{z\cdot\bar z} = \frac{\bar z}{|z|^2} = \bar z$$

$|z|=1.$





$G = \mathbb{Z}\times\mathcal{U} = \{(k,z) : k\in\mathbb{Z},\ z\in\mathcal{U}\}$

\# di giri completi

z posizione sull'anello

$$\begin{cases} (k_1,z_1) \overset{G}{+} (k_2,z_2) = (k_1+k_2(+1),\ z_1\cdot z_2) \\ (k_1,z_1) \overset{G}{\le} (k_2,z_2) \text{ se } k_2>k_1 \text{ oppure se } k_2=k_1 \end{cases}$$

$\begin{cases} \mathrm{Re}\, z_1 \ge \mathrm{Re}\, z_2 & z_1,z_2\in A \\ \mathrm{Re}\, z_1 \le \mathrm{Re}\, z_2 & z_1,z_2\in B \end{cases}$

...

$G,\ \overset{G}{+},\ \overset{G}{\le}$

è un gruppo totalmente ordinato e denso e continuo.

G è isomorfo a $\mathbb{R}$

Scelto $V = (1, 1) \in G$ $\qquad u = 1 \in \mathbb{R}$

angolo giro $\qquad$ 1 giro $\qquad$ "angolo 0"

$\exists!\ f : \mathbb{R} \to G$ t.c.

(i) $f(1) = V$

(ii) $f(x+y) = f(x) \overset{G}{+} f(y)$

(iii) $x \geq 0 \Rightarrow f(x) \overset{G}{\geq} (0,1)$

(iv) $f(0) = (0,1)$

(v) $x \leq y \Rightarrow f(x) \overset{G}{\leq} f(y)$.

$f : \mathbb{R} \to \mathbb{Z} \times \mathcal{U}$ $\qquad \psi : \mathbb{R} \to \mathbb{Z}$, $\varphi : \mathbb{R} \to \mathcal{U}$

$$f(x) = (\psi(x), \varphi(x))$$

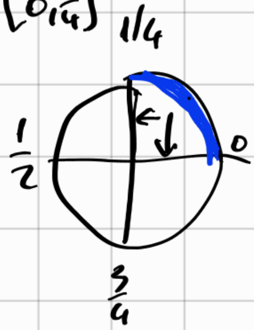
(iv) $\varphi(0) = 1 \in \mathbb{C}$

(i) $\varphi(1) = 1 \in \mathbb{C}$

(ii) $\varphi(x+y) = \varphi(x) \cdot \varphi(y)$

(iii') Im φ è crescente su $[0, \frac{1}{4}]$

Re φ è decrescente



$\cancel{\varphi(x) = \cos x + i \operatorname{sen} x}$

$\cancel{\cos x = \operatorname{Re} \varphi(x).}$

$\cancel{\sin x = \operatorname{Im} \varphi(x).}$

$$\boxed{\text{IN FUTURO} \quad \varphi(x) = e^{2i\pi x}}$$

(iv) $\varphi(x+1) = \varphi(x)$

$$\varphi(x) \cdot \varphi(1) = \varphi(x) \cdot 1$$

Fissato τ la misura dell'angolo. $\tau > 0$

Definiamo $\begin{cases} \cos x = \operatorname{Re} \varphi\left(\frac{x}{\tau}\right) \\ \sin x = \operatorname{Im} \varphi\left(\frac{x}{\tau}\right) \end{cases}$

IN FUTURO $\tau = 2\pi$

$\begin{cases} \cos 0 = 1 \\ \sin 0 = 0 \end{cases}$ $\begin{cases} \cos \tau = 1 \\ \sin \tau = 0 \end{cases}$

$$\varphi\left(\frac{x+y}{\tau}\right) = \varphi\left(\frac{x}{\tau}\right)\cdot\varphi\left(\frac{y}{\tau}\right) = (\cos x + i \operatorname{sen} x)\cdot(\cos y + i \operatorname{sen} y)$$

$$\cos(x+y) + i \operatorname{sen}(x+y) \overset{!}{=} \cos x \cos y - \operatorname{sen} x \operatorname{sen} y + i(\sin x \cos y + \cos x \sin y)$$

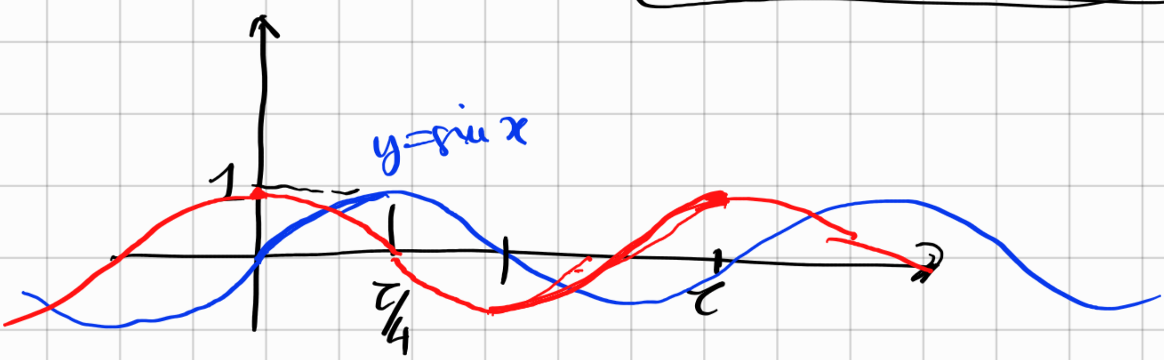
$$\boxed{\begin{cases} \cos(x+y) = \cos x \cos y - \operatorname{sen} x \operatorname{sen} y \\ \operatorname{sen}(x+y) = \sin x \cos y + \cos x \sin y \end{cases}}$$

FORMULE di ADDIZIONE

$\varphi(x) \in U$ $\quad \left|\varphi\left(\frac{x}{\tau}\right)\right|^2 = 1$

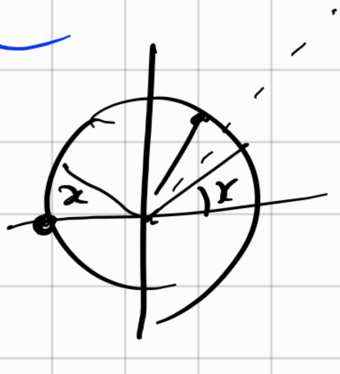
$$\boxed{\cos^2 x + \sin^2 x = 1}$$

FORMULA FONDAMENTALE della TRIGONOMETRIA



$\sin\left(\frac{\tau}{2} - x\right) = \sin x$

$\sin\left(\frac{\tau}{4} - x\right) = \cos x$



$\varphi\left(\frac{1}{2}\right) = -1$