ANALISI MATEMATICA B
LEZIONE 69 - 2.4.2025

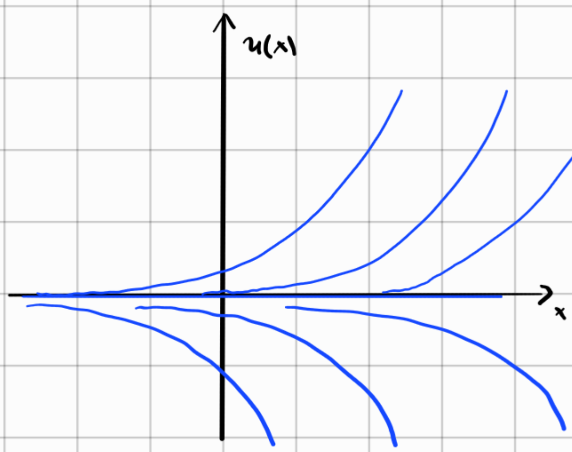
$$u' = u^{\rho}$$

- $\boxed{\rho = 1}$

$$u' = u \implies \int \frac{du}{u} = \int 1\,dx \implies \ln|u| = x + c \implies u = e^{x+c} = e^{c}\cdot e^{x} \implies u = Ke^{x}$$

$$u = -e^{x+c}$$

$$u = 0$$



- $\boxed{\rho > 1}$

$u > 0$ a meno che $\rho \in \mathbb{Z}$

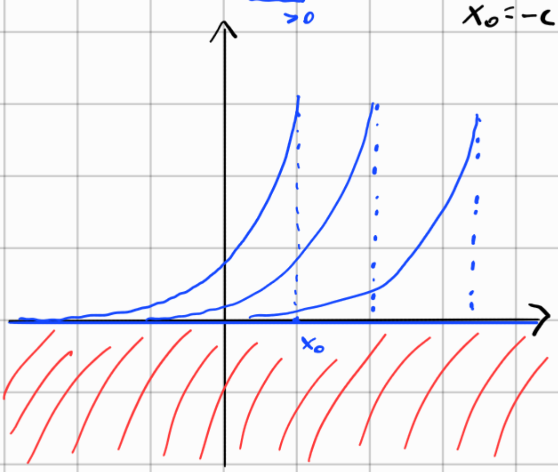
$$u' = u^{\rho} \implies \frac{u'}{u^{\rho}} = 1 \implies \int \frac{du}{u^{\rho}} = x + c \implies \frac{u^{1-\rho}}{1-\rho} = x + c \implies u^{1-\rho} = (x+c)(1-\rho) \implies u = \left((x+c)(1-\rho)\right)^{\overbrace{\frac{1}{1-\rho}}^{<0}}$$

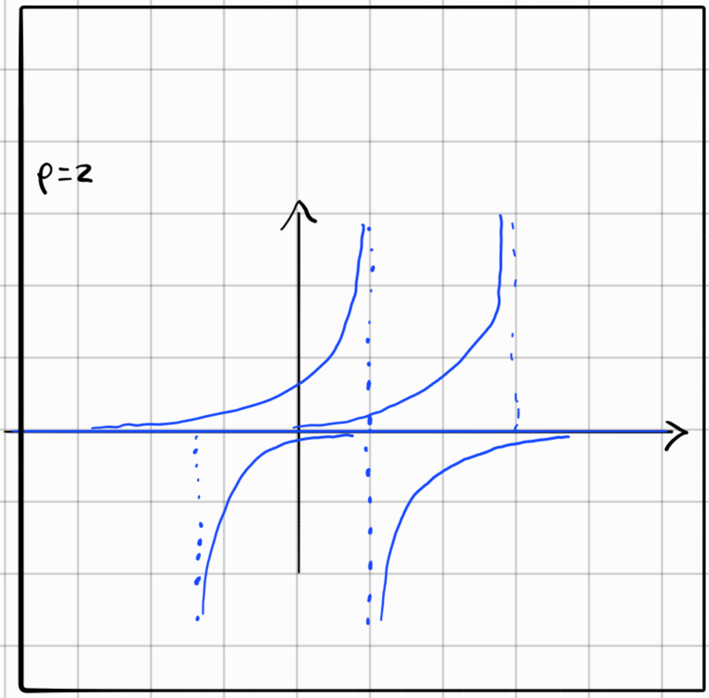
$1-\rho<0$

$x+c<0$
$x<-c$

$$\implies u = \frac{1}{\left(-(x+c)\underbrace{(\rho-1)}_{>0}\right)^{\overbrace{\frac{1}{\rho-1}}^{>0}}}$$

$x_0 = -c$

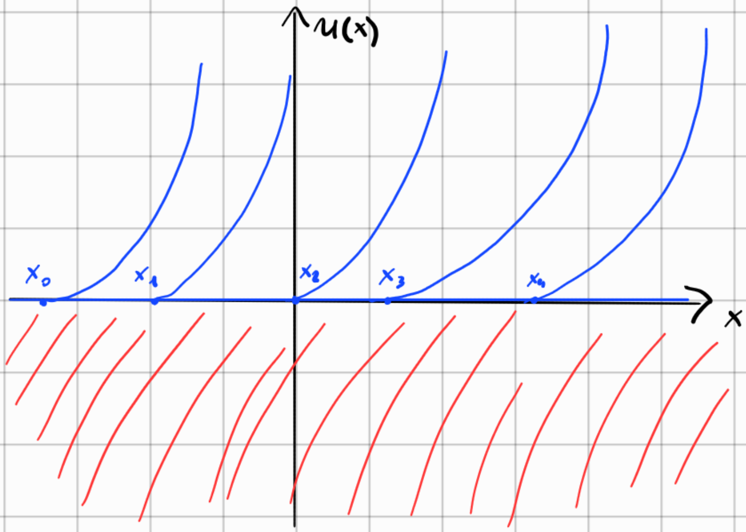


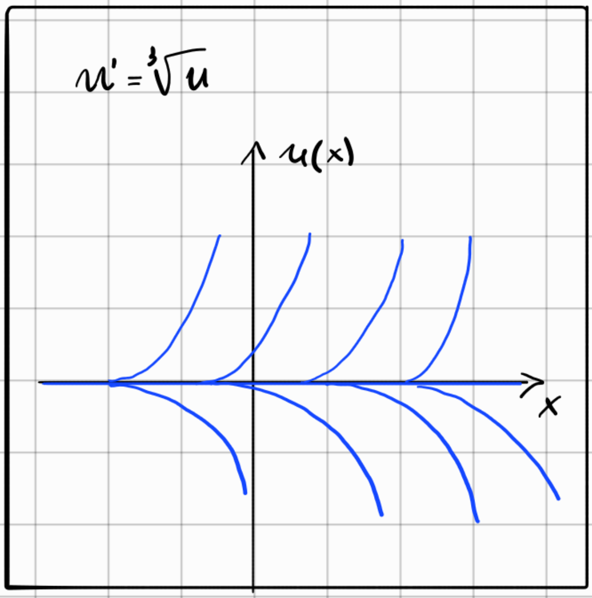


• $\boxed{0<\rho<1}$

$u>0$ a meno che $\frac{1}{\rho}\in\mathbb{Z}$ dispari?

$$u' = u^\rho \Rightarrow \int \frac{du}{u^\rho} = x - x_0 \Rightarrow \underbrace{\frac{u^{1-\rho}}{1-\rho}}_{>0 \Rightarrow x>x_0} = x - x_0 \Rightarrow u = \Big(\overbrace{(x-x_0)(1-\rho)}^{>0}\Big)^{\overbrace{\frac{1}{1-\rho}}^{>1}}$$





$$u' = \sqrt[3]{u^2} \Rightarrow \int \frac{du}{\sqrt[3]{u^2}} = x + c$$

Se $u>0$: $3\sqrt[3]{u} = x + c \Rightarrow u = \left(\frac{x+c}{3}\right)^3$

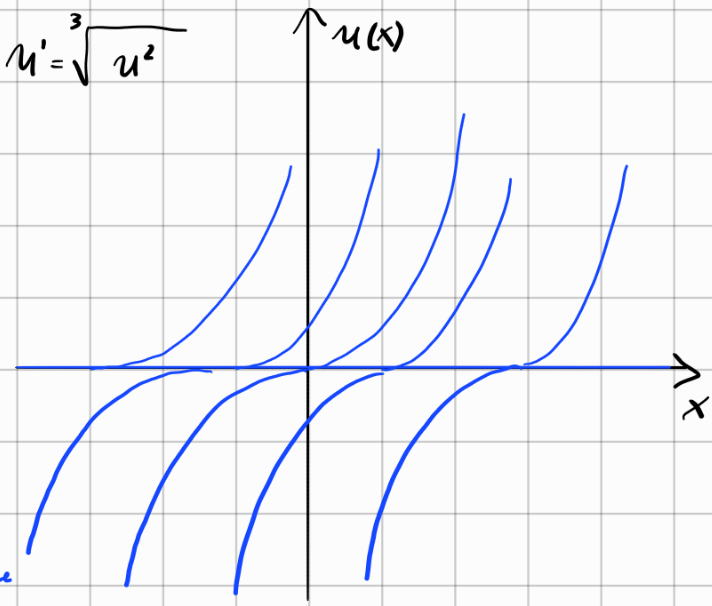
Se $u<0$: $3\cdot\sqrt[3]{u} = x + c \Rightarrow u = \left(\frac{x+c}{3}\right)^3$

$$D\sqrt[3]{x} = \frac{1}{D\, y^3} = \frac{1}{3y^2} = \frac{1}{3(\sqrt[3]{x})^2} = \frac{1}{3\sqrt[3]{x^2}}$$

$$D x^{\frac{1}{3}} \overset{x>0}{=} \frac{1}{3}\cdot x^{-\frac{2}{3}}$$

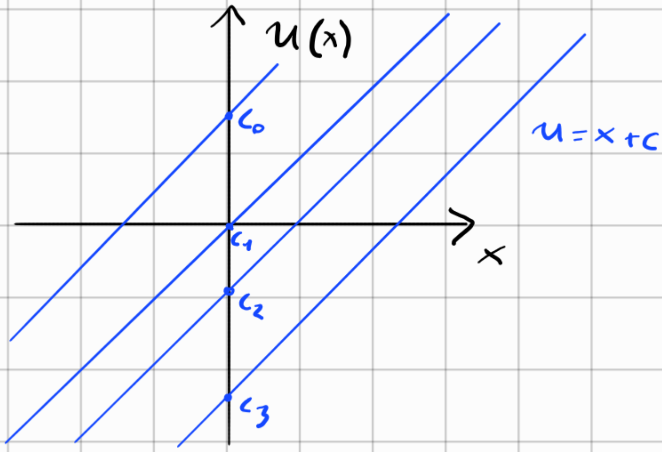
se accetto $x<0$ falliscono alcune proprietà delle potenze

Se $x>0$ le due cose coincidono, se $x<0$ no



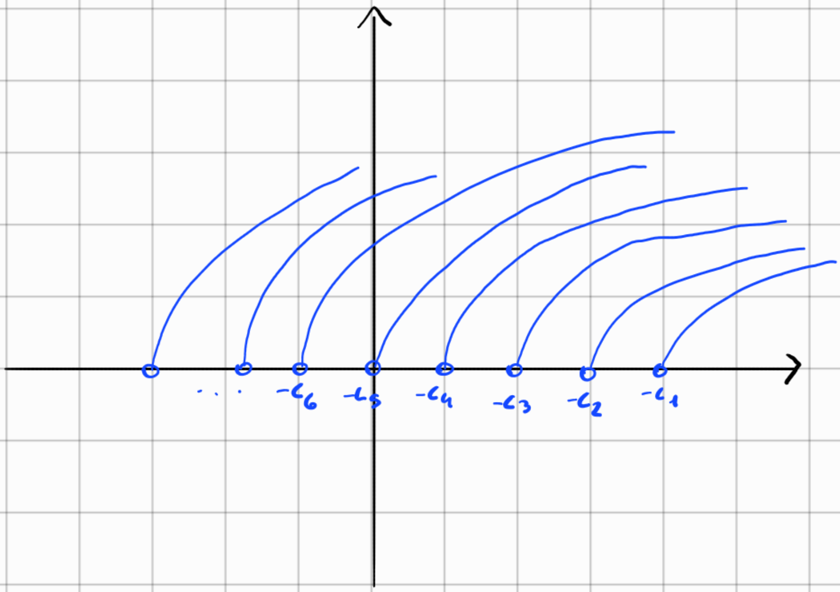
- $\boxed{\rho = 0}$

$$u' = u^0 \underset{\text{se } 0^0 = 1}{=} 1 \quad \Rightarrow \quad u = x + c$$



- $\boxed{\rho < 0}$

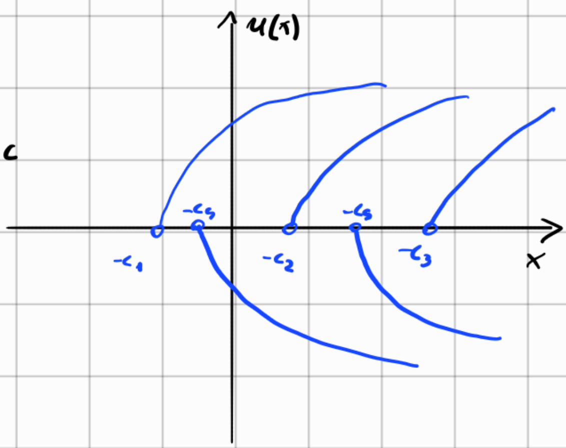
$$\overset{u>0}{u' = u^{\rho}} \Rightarrow \int \frac{du}{u^{\rho}} = x + c \Rightarrow \frac{\overbrace{u^{\overbrace{1-\rho}^{>1}}}^{>0}}{1-\rho} = x + c \quad \overset{x > -c}{\Rightarrow} \quad u = \Big(\overbrace{(x+c)(1-\rho)}^{>0}\Big)^{\overbrace{\frac{1}{1-\rho}}^{0<\ \ <1}}$$



- $\rho = -1$

$$\overset{u \neq 0}{u' = u^{-1}} \Rightarrow u' \cdot u = 1 \Rightarrow \int u\,du = x + c \Rightarrow \frac{u^2}{2} = x + c$$

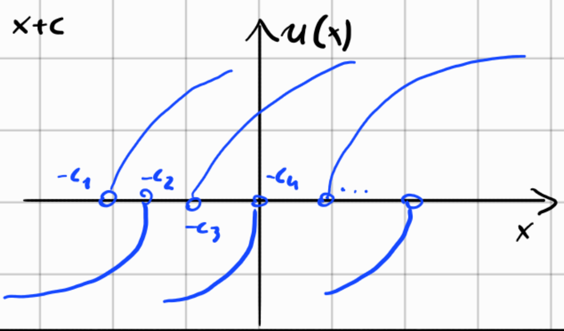
$$\overset{x>-c}{\Rightarrow} u = \pm\sqrt{2x + 2c}$$



- $\rho = -2$

$$\overset{u \neq 0}{u' = u^{-2}} \Rightarrow u' u^2 = 1 \Rightarrow \int u^2\,du = x + c \Rightarrow \frac{u^3}{3} = x + c$$

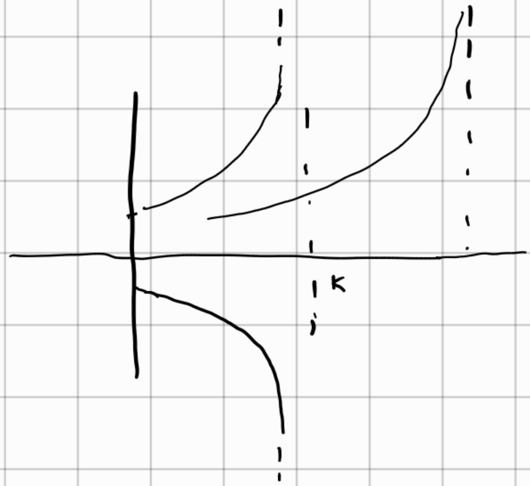
$$\Rightarrow u = \sqrt[3]{3x + 3c}$$



$x \geq 0$

$$u' = u^3 \sqrt{x} \qquad f(x,y) = \sqrt{x}\, y^3 \notin C^1 \; ; \in C^0$$

$$\frac{\partial f}{\partial y} = \sqrt{x}\, 3y^2 \longrightarrow \text{VALE C-L SUL SUO DOMINIO}$$



→ SEPARO VARIABILI

$$\int \frac{1}{u^3} du = \int \sqrt{x} \longrightarrow \frac{-u^{-2}}{2} = \frac{2}{3} x^{\frac{3}{2}} + c$$

$$u^{-2} = -\frac{4}{3} x^{\frac{3}{2}} + c \rightarrow u = \frac{\pm 1}{\left(c - \frac{4}{3} x^{\frac{3}{2}}\right)^{\frac{1}{2}}}$$

$$c - \frac{4}{3} x^{\frac{3}{2}} > 0$$

$$x < K$$

$$u'(x) = u^3(x)\sqrt{x}$$

$$u''(x) = u'(x)\, 3u^2(x)\sqrt{x} + \frac{u^3(x)}{2\sqrt{x}}$$

$$\rightarrow u'' = u^3\sqrt{x}\, 3u^2\sqrt{x} + \frac{u^3}{2\sqrt{x}} = 3u^5 x + \frac{u^3}{2\sqrt{x}}$$

SE $u > 0$, $u'' > 0$ → CONVESSA

SE $u < 0$, $u'' < 0$ → CONCAVA