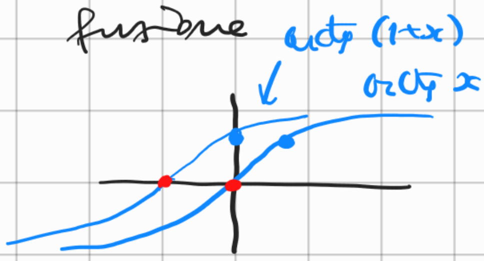
ANALISI MATEMATICA B
LEZIONE 35 - 16.2.2026

II COMPITINO 20.2.2021

1.(a) Scrivere il polinomio di Taylor di ordine 2 centrato in $x_0 = 0$ per la funzione

$$\operatorname{arctg}(1+x)$$



(b) Calcolare

$$\lim_{x\to 0} \frac{\cos(\sqrt{2}x) - \sqrt{2}\sin(\operatorname{arctg}(\cos(2x)))}{(e^x + e^{-x})^5 - 32 - 80x^2}$$

$\operatorname{arctg}(1+0)$

$$P(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2$$

$f(x) = \operatorname{arctg}(1+x)$

$f(0) = \frac{\pi}{4}$

$$f'(x) = \frac{1}{1+(1+x)^2}$$

$f'(0) = \frac{1}{2}$

$$f''(x) = -\frac{2x+2}{(1+(1+x)^2)^2}$$

$f''(0) = -\frac{1}{2}$

$$P(x) = \frac{\pi}{4} + \frac{1}{2}x - \frac{1}{4}x^2$$

EVITIAMO QUESTO ERRORE:

$$\operatorname{arctg} x = x + o(x^3) \qquad \text{per } x \to 0$$

$$\operatorname{arctg}(1+x) = (1+x) + o\left((1+x)^3\right) \qquad \text{per } \cancel{x \to 0}$$

NO!

$$\boxed{\operatorname{arctg}(1+t) = \frac{\pi}{4} + \frac{1}{2}t - \frac{1}{4}t^2 + o(t^2) \quad \text{per } t \to 0}$$

$1+x \to 0$

cioè $x \to -1$

$$\sin\left(\operatorname{arctg}(\cos 2x)\right) = \sin\left(\operatorname{arctg}\left(1 - \frac{(2x)^2}{2} + \frac{(2x)^4}{4!} + o(x^4)\right)\right) \qquad (x \to 0)$$

$$\operatorname{arctg}(1+t) = \frac{\pi}{4} + \frac{1}{2}t - \frac{1}{4}t^2 + o(t^2) \qquad (t \to 0)$$

$$t = -\frac{(2x)^2}{2} + \frac{(2x)^4}{4!} + o(x^4)$$

$$\frac{1}{2}t = -\frac{1}{2}\,\frac{4}{2}x^2 + \frac{1}{2}\,\frac{16x^4}{4\cdot 3\cdot 2} + o(x^4) = -x^2 + \frac{1}{3}x^4 + o(x^4)$$

$$-\frac{1}{4}t^2 = -\frac{1}{4}\left(-2x^2 + \frac{16}{4\cdot 3\cdot 2}x^4 + o(x^4)\right)^2$$

(*)

$$= -\frac{1}{4}\left(4x^4 + o(x^4)\right) = -x^4 + o(x^4)$$

$$\operatorname{arctg}(1+t) = \frac{\pi}{4} - x^2 + \frac{1}{3}x^4 - x^4 + o(x^4)$$

$$= \frac{\pi}{4} - x^2 - \frac{2}{3}x^4 + o(x^4)$$

$$S = x^2 + \frac{2}{3}x^4 + o(x^4)$$

$$\sin\left(\frac{\pi}{4} - S\right) = \sin\frac{\pi}{4}\cos S - \cos\frac{\pi}{4}\sin S$$

$$\textcircled{*} \quad (A+B+C)^2 = A^2 + 2AB + B^2 + 2AC + 2BC + C^2$$

$$\left(-2x^2 + \frac{16}{4\cdot 3\cdot 2}x^4 + o(x^4)\right)^2 = (-2x^2)^2 + \square\, x^2 x^4 + (\heartsuit x^4)^2 + \square\,(-2x^2\cdot o(x^4)) + \square\,(x^4)^2 + \dots$$

$$= 4x^4 + \square\, x^6 + o(x^6)$$

$$= 4x^4 + o(x^4)$$

$$\sin\left(\frac{\pi}{4} - S\right) = \frac{\sqrt{2}}{2}\cos\left(x^2 + \frac{2}{3}x^4 + o(x^4)\right) - \frac{\sqrt{2}}{2}\sin\left(x^2 + \frac{2}{3}x^4 + o(x^4)\right)$$

$$\cos\left(x^2 + o(x^2)\right) = 1 - \frac{(x^2 + o(x^2))^2}{2} + o\left((x^2 + o(x^2))^2\right)$$

$$= 1 - \frac{x^4}{2} + o(x^4)$$

$$\boxed{\cos t = 1 - \frac{t^2}{2} + o(t^2)}$$

$$\sin t = t - \frac{t^3}{6} + o(t^4)$$

$$\sin\left(x^2 + \frac{2}{3}x^4 + o(x^4)\right) =$$

$$= x^2 + \frac{2}{3}x^4 + o(x^4) - \frac{1}{6}\cancel{\left(x^2 + o(x^2)\right)^3} + \cancel{o\left(x^6\right)}$$

$$= x^2 + \frac{2}{3}x^4 + o(x^4)$$

$$\sin\left(\frac{\pi}{4} - S\right) = \frac{\sqrt{2}}{2}\left[1 - \frac{x^4}{2} - x^2 - \frac{2}{3}x^4\right] + o(x^4)$$

$$\frac{1}{2} + \frac{2}{3} = \frac{3+4}{6} = \frac{7}{6}$$

$$= \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}x^2 - \frac{\sqrt{2}}{2}\left(\frac{1}{2} + \frac{2}{3}\right)x^4 + o(x^4)$$

$$N(x) = \cos\left(\sqrt{2}x\right) - \sqrt{S}\sin\left(\frac{\pi}{4} - S\right)$$

$$= \cancel{1} - \cancel{\frac{(\sqrt{2}x)^2}{2}} + \frac{(\sqrt{2}x)^4}{4!} - \sqrt{2}\left[\cancel{\frac{\sqrt{2}}{2}} - \cancel{\frac{\sqrt{2}}{2}x^2} - \frac{\sqrt{2}\cdot 7}{12}x^4\right] + o(x^4)$$

$$= \frac{x^4}{6} + \frac{7}{6}x^4 + o(x^4) = \frac{4}{3}x^4 + o(x^4)$$

11
121
1331
14641

$$\left(e^x + e^{-x}\right)^5 \qquad (a+b)^5 = 1\,5\,10\,10\,5\,1$$

$$\left(e^x + e^{-x}\right)^5 = \left(2\cosh x\right)^5 = 2^5\Big(\underbrace{1 + \frac{x^2}{2} + \frac{x^4}{4!} + o(x^4)}_{a+b}\Big)^5$$

$$= 32\left(1 + 5\left(\frac{x^2}{2} + \frac{x^4}{4!} + o(x^4)\right) + 10\left(\frac{x^2}{2} + o(x^2)\right)^2 + o(x^4)\right)$$

$$= 32\left(1+\frac{5}{2}x^2+\frac{5}{24}x^4+\frac{10}{4}x^4+o(x^4)\right)$$

$$= 32 + 80x^2 + 32\left(\frac{5}{24}+\frac{60}{24}\right)x^4 + o(x^4)$$

$$= 32 + 80x^2 + \frac{4}{3}\cdot 65\, x^4$$

130

$$= 32 + 80x^2 + \frac{260}{3}x^4$$

$$\lim_{x\to 0}\frac{0}{0} = \frac{\frac{4}{3}}{\frac{260}{3}} = \frac{1}{65}$$

---

3. $$f(x) = \cos x + \ln(1-\sin x) - e^{-x} + \frac{3}{2}\operatorname{tg}(x^2+x^4)$$

(a) Dire se $x_0=0$ è max o min locale

(b) Determinare P. di Taylor di ordine 3 centrato in $x_0=0$ per la funzione $f'(x)$.

(c) $$\sum (-1)^n \Big(\underbrace{f\left(\tfrac{1}{n}\right)}_{a_n}\Big)^d$$ per quali d converge?

$$\boxed{f(x) = 0 + c\,x^n + o(x^n)}$$

TROVATA QUESTA POSSO RISPONDERE A TUTTO.

$$= x^n(c + o(1))$$

(a) n — pari — $c>0$ ......
— $c<0$ ......
— dispari ......

(b) $$\boxed{f'(x) = \underbrace{n c x^{n-1}} + o(x^{n-1})}$$

↑ è il P. di Taylor di ordine $n-1$

(c) $n=4$, $c>0$, $$f\left(\tfrac{1}{n}\right) = \frac{1}{n^4}\left(c+o(1)\right)$$

$$f^{\alpha}\left(\tfrac{1}{n}\right) = \frac{1}{n^{4\alpha}}\left(c+o(1)\right)^{\alpha}$$

$$\sim \frac{c^{\alpha}}{n^{4\alpha}} = \frac{k}{n^{p}} \qquad p = 4\alpha$$

converge assolutamente $\iff p>1$ $\qquad \alpha > \frac{1}{4}$

converge semplicemente? Valgono le ipotesi di Leibniz?

(1) $f\left(\frac{1}{n}\right)^{\alpha} \to 0$? $\quad \frac{1}{n^{4\alpha}} \quad 4\alpha > 0 \quad \underline{\underline{\alpha>0}}$

(2) $\left(f\left(\frac{1}{n}\right)\right)^{\alpha}$ decrescente?

$\Updownarrow$

$f\left(\frac{1}{n}\right)$ decrescente?

$\Uparrow$

$x = \frac{1}{n}$ $\qquad f'(x) > 0$ per x in un intorno destro di 0.

$$f'(x) = 3c x^3 + o(x^3) = x^3\left(3c + o(1)\right) > 0$$

↑ $c>0$ $\qquad$ se $x \in (0,\varepsilon)$.