ANALISI MATEMATICA B
LEZIONE 68 - 20.3.2026

Studio qualitativo

$$u' = u^2 + x$$

Vale teo di $\exists$!

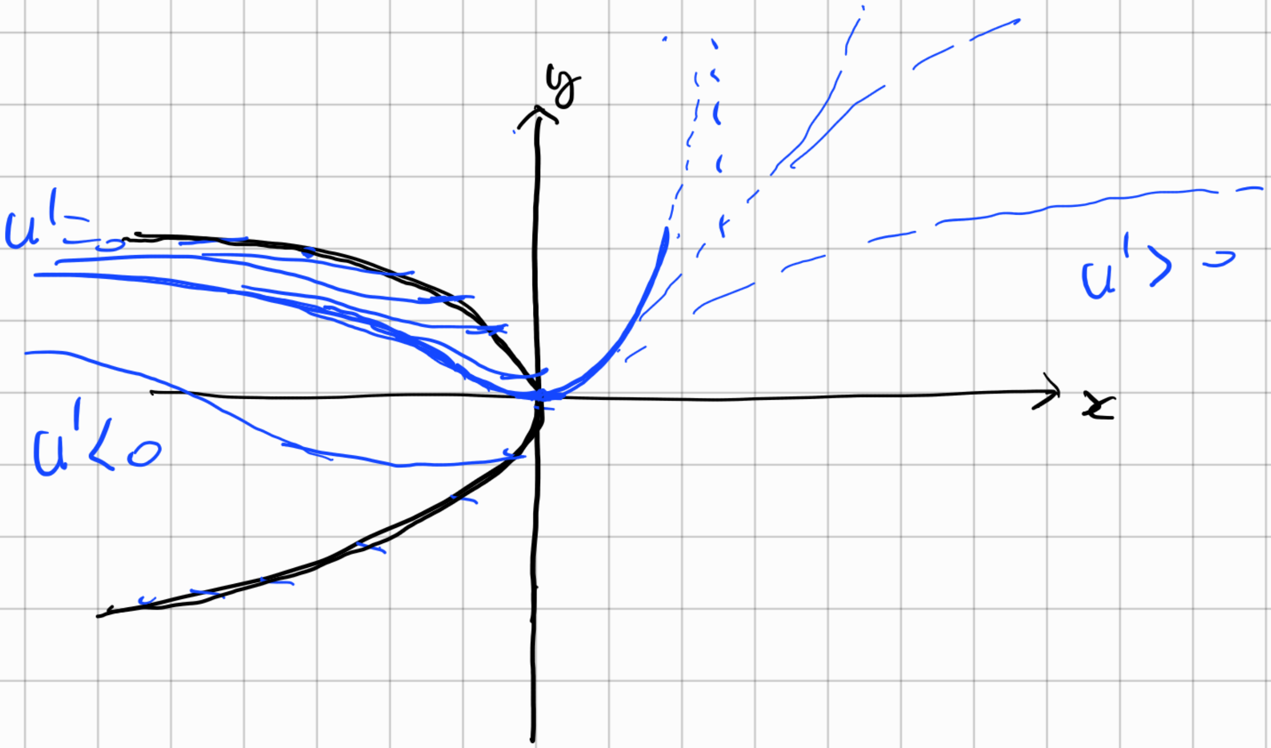
come sono fatte le soluzioni?

Studio il segno di $u' = u^2 + x$ $\qquad y = u(x)$

$u' = 0$ se $u^2 = -x$ $\qquad y = u(x)$ $\qquad x = -y^2$

$u' > 0$ se $u^2 + x > 0$ $\qquad x > -y^2$

$u' < 0$ $\qquad x < -y^2$

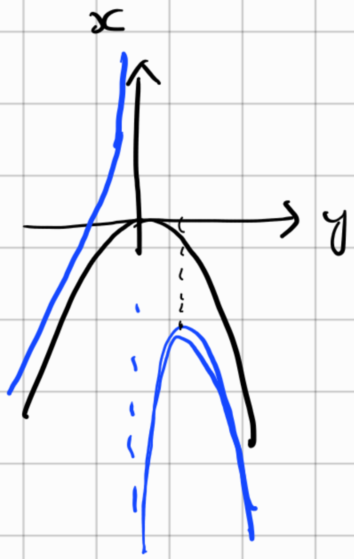


Potrei studiare anche il segno di u':

$$u'' = (u^2 + x)' = 2uu' + 1 = 2u(u^2 + x) + 1$$

$$u'' = 2u^3 + 2ux + 1$$

$$u'' = 0 \qquad 2y^3 + 2xy + 1 = 0$$



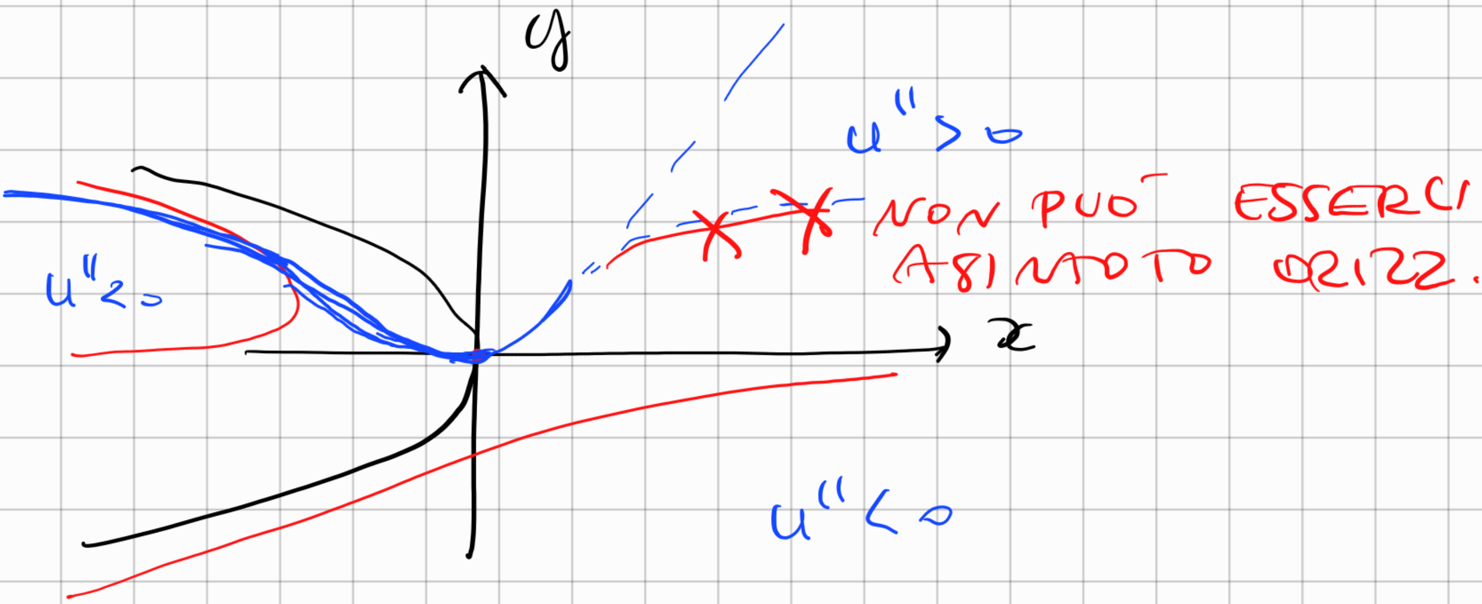
$$x = \frac{-1-2y^3}{2y} = -y^2 - \frac{1}{2y}$$

$$x = g(y)$$

$$g'(y) = -2y + \frac{1}{2y^2} = 0$$

$$2y = \frac{1}{2y^2}$$

$$4y^3 = 1$$



METODO ALTERNATIVO

se u avesse un asintoto orizzontale

$$u \to \ell \qquad \text{per } x \to +\infty$$

$$u' = u^2 + x \longrightarrow \ell^2 + \infty = +\infty$$

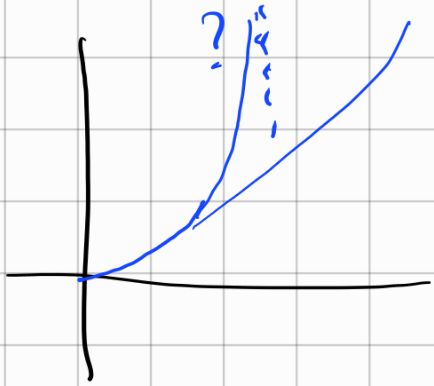
per $x \to +\infty$

$$\text{Se} \quad \left.\begin{array}{l} u \longrightarrow \ell \in \mathbb{R} \\ u' \longrightarrow m \in \overline{\mathbb{R}} \end{array}\right\} \Rightarrow m = 0.$$

ASSURDO

C'E' UN ASINTOTO VERTICALE

O C'E' ESISTENZA GLOBALE



$u' = u^2 + x$

NON SI APPLICA IL TEO DI ESISTENZA GLOBALE

$u' = f(x,u)$

$$|f(x,y)| \leq m|y| + q$$

ma $f(x,y) = y^2 + x$ non è sublineare!

$u' = u^2 + x > u^2 \quad \forall x > 0$

ma $u' = u^2$ ($u' = u^p$ con $p > 1$)

ha asintoto verticale

CRITERIO DI CONFRONTO

$$\begin{cases} u_1'(x) = f_1(x, u_1(x)) \\ u_1(x_0) = y_1 \end{cases} \qquad \begin{cases} u_2'(x) = f_2(x, u_2(x)) \\ u_2(x_0) = y_2 \end{cases}$$

se $y_1 > y_2$ e $\boxed{f_1(x, y) > f_2(x, y)}$ $\forall x > x_0 \quad \forall y \in \mathbb{R}$

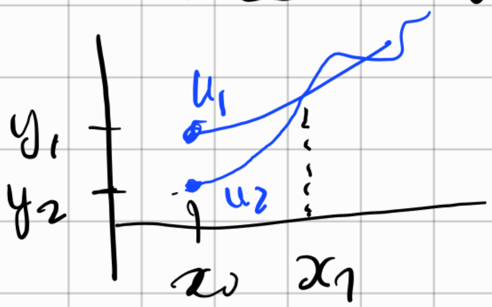
allora $u_1(x) > u_2(x) \quad \forall x > x_0$.

dim se per assurdo $\exists x > x_0$ t.c.

$$u_2(x) > u_1(x)$$

$\exists x_1$ primo punto in cui $u_2(x_1) = u_1(x_1)$

cioè $\forall x \in [x_0, x_1] \quad u_2(x) < u_1(x)$

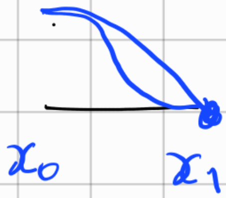


Allora $u_1'(x_1) \leq u_2'(x_1)$

$$\underbrace{u_1'(x_1)}_{f_1(x_1, u_1(x_1))} \qquad f_1(x_1, u_1(x_1)) > f_2(x_1, u_2(x_1))$$

ASSURDO!!

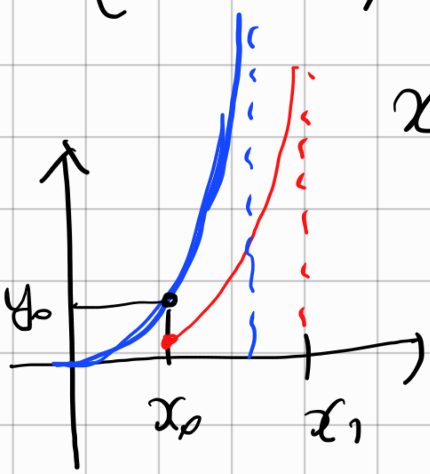
$\Big(u_1 - u_2$ in $[x_0, x_1]$ ha minimo in x_1 $\Rightarrow (u_1 - u_2)'(x_1) \le 0 \Big)$



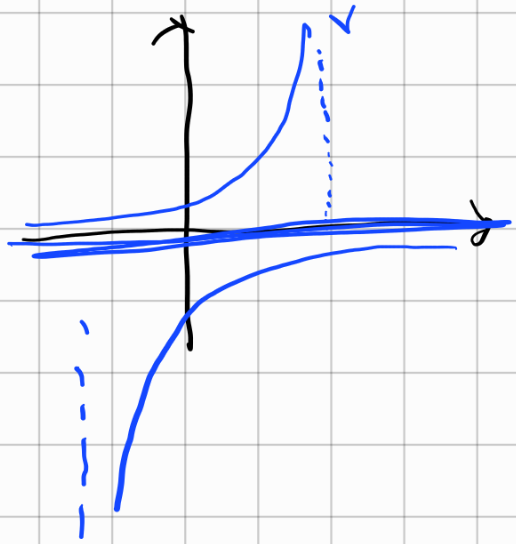
$\square$

Nel nostro caso

$$\begin{cases} u' = u^2 + x \\ u(x_0) = y_0 > 0 \end{cases} \qquad \begin{cases} v' = v^2 \\ v(x_0) = \frac{y_0}{2} \end{cases}$$

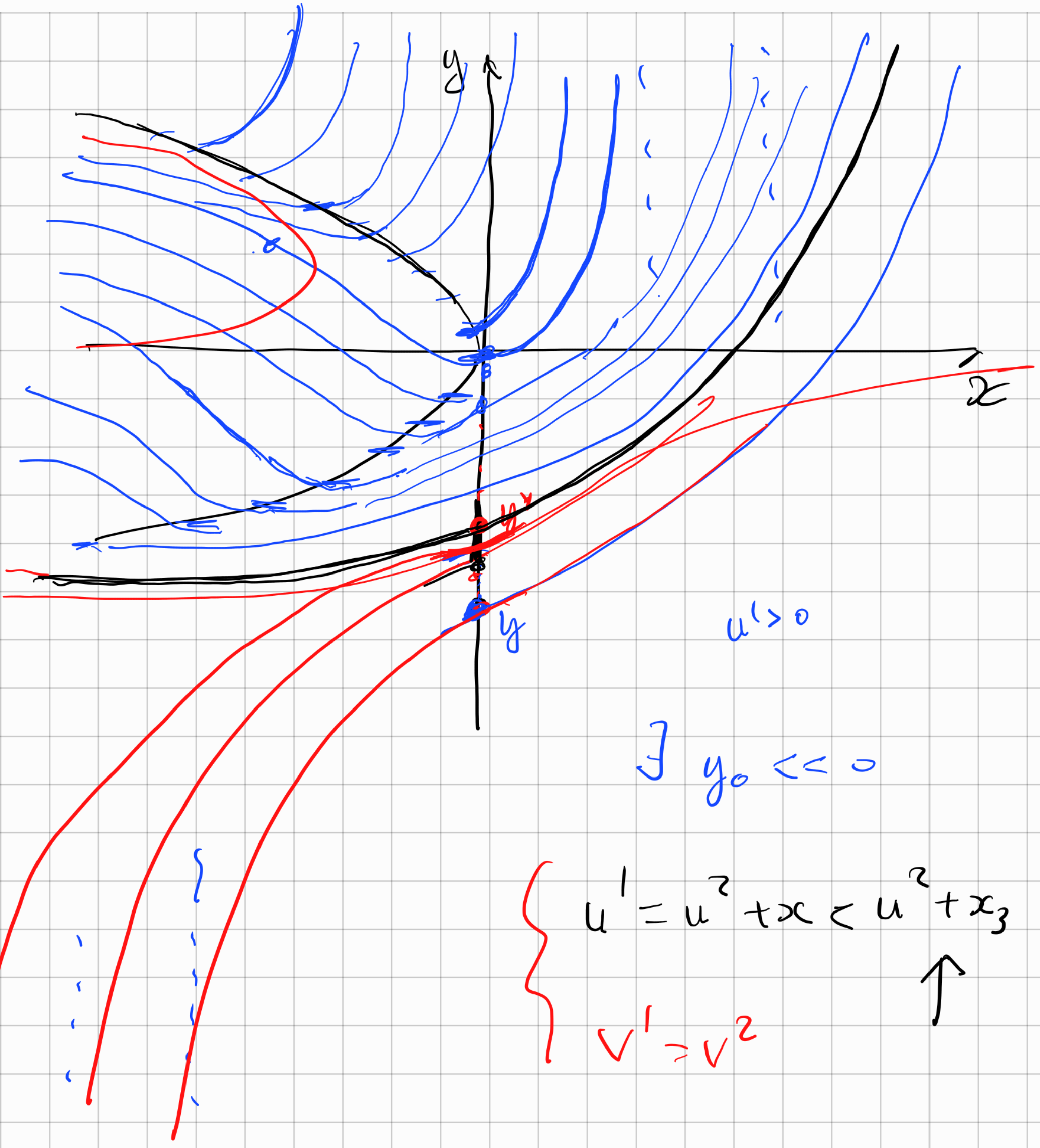


$x_0 > 0$



$$\Big|\ \begin{array}{l} y^2 + x > y^2 \\ \text{per } x \geq x_0 \end{array}$$

dunque $u(x) > v(x)$ ha un asintoto in $x_1 > x_0$



$\exists\, y_0 << 0$

$$\begin{cases} u' = u^2 + x < u^2 + x_3 \\ v' = v^2 \end{cases}$$

$A = \{ y \in \mathbb{R} : y < 0 :$ la soluzione che passa per $(0, y)$ tocca la parabola $\}$

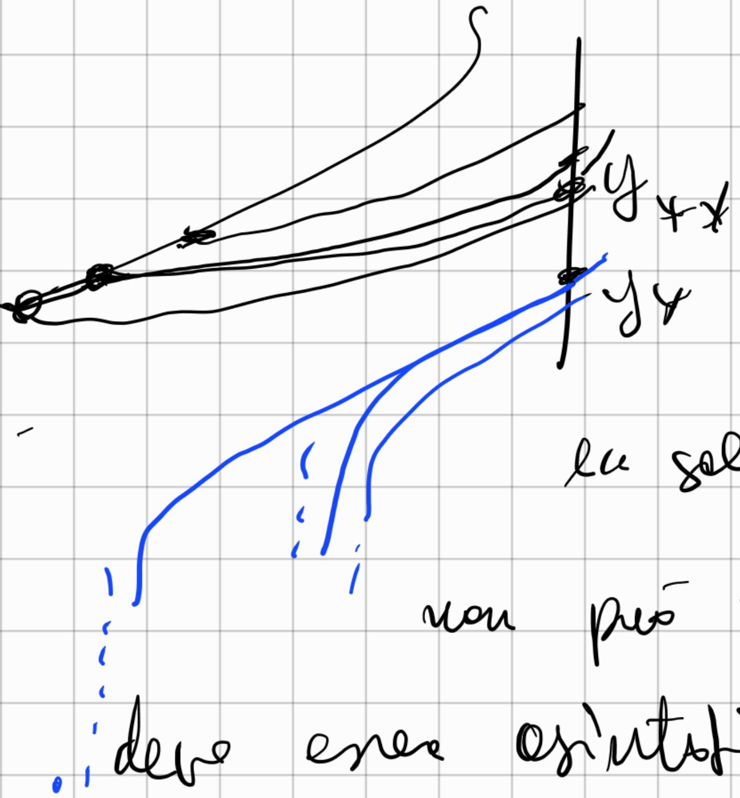
$B = \{$ la soluzione ha un asintoto $\}$

$A \leq B$

c'è almeno un punto di separazione!

se ce ne fossero 2 prendo $y_{**} = \inf A$

$y_* = \sup B$



la soluzione u_{**} che parte da y^{**}

non può toccare la parabola

deve essere asintotica alla parabola

Bisogna trovare una regione di piano nel III quadrante da cui tutte le soluzioni hanno un asintoto.