Alg. Lin. 16/12/2015

$\boxed{40}$ (1) $\mathbb{R}^3$ V, W

$V: \quad \boxed{5 \overset{4}{x_1} - 4 \overset{5}{x_2} + \cancel{6 x_3}} = 0$

$W: \quad 9 x_1 + 7 x_2 - 6 x_3 = 0$

(A) $\mathcal{B}$ base di V

$$\begin{pmatrix} 4 \\ 5 \\ 0 \end{pmatrix}, \begin{pmatrix} 6 \\ 0 \\ -5 \end{pmatrix}, \cancel{\begin{pmatrix} 0 \\ 6 \\ 4 \end{pmatrix}} \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}$$

no

diviso per 2

perché 6 e 4 non sono primi tra loro

$$v_1 = \begin{pmatrix} 4 \\ 5 \\ 0 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}$$

(B) $\underbrace{\begin{pmatrix} -7 \\ 9 \\ 0 \end{pmatrix}}_{\ker}, \cancel{\begin{pmatrix} 6 \\ 0 \\ 9 \end{pmatrix}} \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 6 \\ 7 \end{pmatrix}$

$$\mathcal{C} = \{w_1, w_2\} \quad w_1 = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}, \quad w_2 = \begin{pmatrix} 0 \\ 6 \\ 7 \end{pmatrix}$$

(c) $f(\underbrace{8e_1 + e_2 - 6e_3}_{v})$ date $[f]_B^{\mathcal{E}}$

$$8e_1 + e_2 - 6e_3 = \alpha v_1 + \beta v_2$$

$$\begin{pmatrix} 8 \\ 1 \\ -6 \end{pmatrix} = \alpha \begin{pmatrix} 4 \\ 5 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}$$

$\alpha = 2$

$\beta = -3$

$$[v]_B = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} -2 & 5 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 2 \\ -3 \end{pmatrix} = \begin{pmatrix} -19 \\ -7 \end{pmatrix}$$

$$[f]^{\mathcal{C}}_{\mathcal{B}} \, [v]_{\mathcal{B}} = [f(v)]_{\mathcal{C}}$$

$$f(v) = -19 \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} - 7 \begin{pmatrix} 0 \\ 6 \\ 7 \end{pmatrix} = \begin{pmatrix} -38 \\ -42 \\ -106 \end{pmatrix}$$

(D) $g(x) = \begin{pmatrix} 23 & -20 & 24 \\ 3 & 0 & -6 \\ 38 & -30 & 29 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ definisce $g : V \to W$

$$\begin{pmatrix} 23x_1 - 20x_2 + 24x_3 \\ 3x_1 - 6x_3 \\ 38x_1 - 30x_2 + 29x_3 \end{pmatrix} \overset{?}{\in} W \qquad \underbrace{5x_1 - 4x_2 + 6x_3 = 0}_{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in V}$$

cioè

$$9(23x_1 - 20x_2 + 24x_3) + 7(3x_1 - 6x_3) +$$
$$-6(38x_1 - 30x_2 + 29x_3) \overset{?}{=} 0$$

$$207 x_1 - \cancel{180} x_2 + 216 x_3 + 21 x_1 - 42 x_3 +$$

$$-228 x_1 + \cancel{180} x_2 - 174 x_3 \stackrel{?}{=} 0$$

$$0 x_1 + 0 x_2 + 0 x_3 = 0 \quad \underline{\text{ok}}$$

$$[g]_B^{\mathcal{C}} = \left(\quad \right)$$

$$g(v_1) = \alpha w_1 + \beta w_2$$

$$\begin{cases} \_\_\_\_ = \alpha \_ + \beta \_\_ \\ \_\_\_\_ = \alpha \_ + \beta \_\_ \\ \_\_\_\_ = \alpha \_ + \beta \_\_ \end{cases}$$

una delle 3 eq dipende dalle altre 2

($\Rightarrow$ la cancello) ricavo trovando

α e β

$\boxed{1}$ (1) (c)

$$\begin{cases} 4x + y + 2z = 3k \\ 3x + (k+3)y - z = 4 \\ kx + 3y - 7z = +17 \end{cases}$$

$k_0 = -1$

W = giacitura dello sp. a. sol

so che per $k = -1$ $\dim W = 1$

$$\begin{cases} 4x + y + 2z = -3 \\ 3x + 2y - z = 4 \end{cases}$$ sono indipendenti

$-x + 3y - 7z = 17$ ⟵ posso eliminarla

$$Z = \operatorname{Span}\left(\begin{pmatrix} 2 \\ 1 \\ 7 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} \right)$$

$$\mathbb{R}^3 = W \oplus Z$$

determinare la matrice delle proiezioni

Cerco un generatore di W

$$w = \begin{pmatrix} \cancel{-5} \\ \cancel{+10} \\ \cancel{5} \end{pmatrix} \quad \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$$

$$\det \begin{pmatrix} 1 & -1 \\ 7 & 3 \end{pmatrix}$$

Cerco equazione per Z

$$\cancel{10x + 8y - 4z = 0}$$

$$5x + 4y - 2z = 0$$

$$\mathbb{R}^3 \ni v = p(v) + q(v)$$

$$p : \mathbb{R}^3 \to W$$

$$q : \mathbb{R}^3 \to Z$$

$(W \cap Z = \{0\}$ $\quad \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$ non annulla $5x+4y-2z)$

risultato -1

$$v = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \underset{\lambda w}{\underset{\shortparallel}{p(v)}} + \underset{\in Z}{q(v)}$$

calcolo l'eq. di Z

a 5x e dx

$$5x + 4y - 2z = -\lambda + 0$$

$$\lambda = -5x - 4y + 2z$$

$$p(v) = (-5x - 4y + 2z)\begin{pmatrix} +1 \\ -2 \\ -1 \end{pmatrix}$$

$$= \begin{pmatrix} -5 & -4 & 2 \\ 10 & 8 & -4 \\ 5 & 4 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

matrice di P

matrice di ρ è data

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} \text{matrice} \\ \text{di } p \end{pmatrix}$$

$\boxed{3}$ (2)

$$E_k = \underbrace{\begin{pmatrix} 2 \\ 0 \\ k-4 \end{pmatrix}}_{=v_0} + \operatorname{Span}\left(\underbrace{\begin{pmatrix} 2 \\ k+5 \\ 2 \end{pmatrix}}_{=v_1}, \underbrace{\begin{pmatrix} k \\ 2k \\ -1 \end{pmatrix}}_{=v_2} \right)$$

$$F_h = \begin{cases} x + (h+3)y + (h+1)z = 1 \\ 3h\,x + 12y + 2(5-2h)z = h+2 \end{cases}$$

(A) Trovare n_0, n_1, k_0 t.c.

$$\dim(E_{k_0}) = n_0 \qquad \dim(E_k) = n_1 \quad k \neq k_0$$

$$\operatorname{rank}\begin{pmatrix} 2 & k \\ k+5 & 2k \\ 2 & -1 \end{pmatrix} \begin{cases} 2 \\ 1 \quad \leftarrow \text{per } k = k_0 \end{cases}$$

$n_0 = 1 \qquad n_1 = 2$

trovo k_0 risolvendo $rk\left(\qquad \right) = \underline{\underline{1}}$

$(E) \qquad F_{(-2)} \qquad \dim F_{(-2)} = 1$

posizione reciproca di $F_{(-2)}$ e $E_{(k)}$

$E_k \cap F_{(-2)} = \emptyset \qquad$ trovo descriz parametrica di $F_{(-2)}$

$$F_{(-2)} = u_0 + \mathrm{Span}(w)$$

$$E_k \cap F_{(-2)} = \emptyset \quad k$$

$$\boxed{v_0 - u_0 \notin \mathrm{Span}(v_1, v_2, w)}$$

↳ giacitura di $F_{(-2)} \subset$ giacitura di E

$$\mathrm{rank}(v_1, v_2, w) < \mathrm{rank}(v_0 - u_0, v_1, v_2, w)$$

2) $\cdot$ 3

k $\quad rk(v_1, v_2, w) = 3 \Rightarrow$

si avrà che $E_k \cap F_{(-2)} \neq \emptyset$

$- \; E_k \cap F_{(-2)} = \emptyset$

$- \; E_k \cap F_{(-2)} \neq \emptyset$ $\quad \dim(E_k \cap F_{(2)}) =$

$\dim$ (giacitura $E_k \cap$ giac. $F_{(2)}$)