

Teorie dei nodi 15/12/16

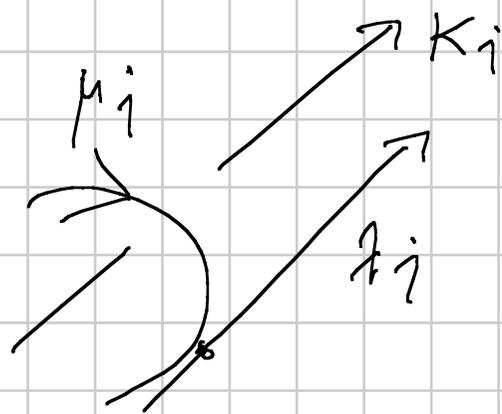
LCS³ link $L = K_1 U \dots U K_m$, $\alpha_j \subset \partial U(K_j)$ c.s.c. non-triviali
chirurgie $(\mathbb{D}^3, U(L)) \cup_{f_1, \dots, f_m} (m \cdot (\mathbb{D}^2 \times S^1))$

$$f_j: \partial \mathbb{D}^2 \times \{*\} = \alpha_j$$

Fatto: α_j / isotopia determinata da $\pm [\alpha_j] \in H_1(\partial U(K_j))$

Se f_j longitudinale preferita e μ_j meridiano, con

$$f_j, \mu_j \text{ positive, } \pm [\alpha_j] = \pm (a_j \mu_j + b_j \cdot f_j) \quad \text{con } a_j, b_j \text{ coprimi}$$

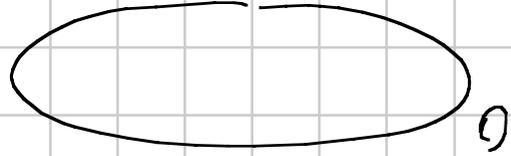


è ben def $a_{ij}/b_{ij} \in \mathbb{Q} \cup \{\infty\}$
indip. de orientaz. di K_i

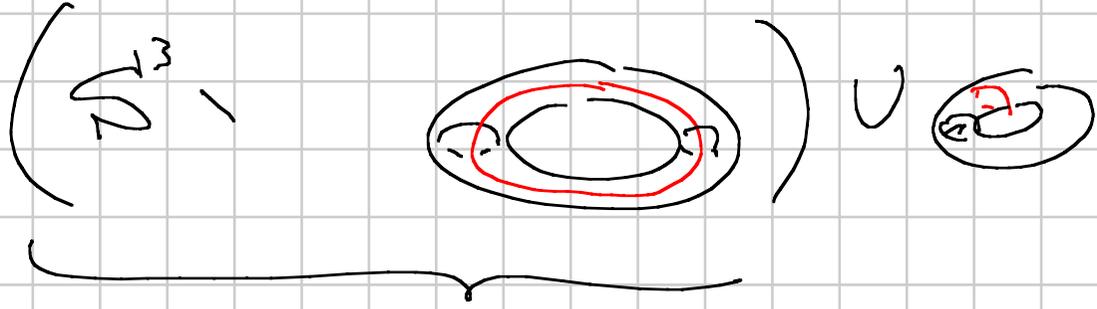
Morale: chirurgia su L determinata da $g_1, \dots, g_n \in \mathbb{Q} \cup \{\infty\}$

Oss: con ∞ la chirurgia non ha effetti.

Es:

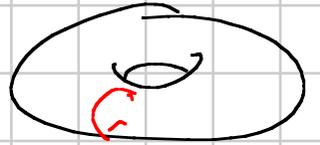
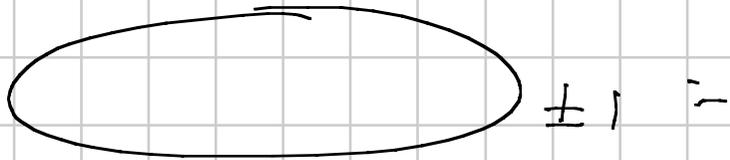


"



" $\partial^2_x \times \partial^2_y$

ES:



$$= \mathbb{S}^3$$

Ricordo: attaccare j -manico a $M^{(m)}$ con ∂ .

$$f: \partial D^i \times D^{m-j} \rightarrow \partial M$$

$$M' = M \cup_f (D^i \times D^{m-j})$$

$$M = \mathbb{R}^3$$

$$i = 0$$

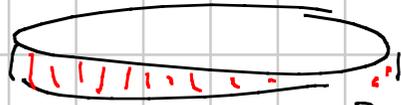
$$M \xrightarrow{\quad} M' = M \cup \text{circle}$$

$$i = 1$$



$$i = 2$$

altrimenti



$$i = 3$$

appare un buco \mathbb{D}^2

oss: $\partial M' = \partial M - f(\partial \mathbb{D}^i \times \mathbb{D}^{m-i}) \cup f|_{\partial \mathbb{D}^i \times \partial \mathbb{D}^{m-i}} \mathbb{D}^i \times \partial \mathbb{D}^{m-i}$

$$m=4, M = D^4$$

$$f: \partial D^2 \times D^2 \rightarrow S^3 \Rightarrow \text{Im } f = \text{toro solido intorno del nodo } \partial D^2 \times \{0\}$$

$$\partial M' = (S^3 \setminus \text{Im } f) \cup_f (D^2 \times S^1)$$

$\rightarrow \bar{e}$ è una chirurgia e pu costruirsi
 \bar{e} intesa ($f(S^1 \times \{x\})$ è longitudine)

Conseguenza: rappresenta una M^3 classe orientata

come chirurgia intorno lungo link in \mathbb{S}^3 equivale a esprimere M con $\partial(D^4 \text{ con attaccati } 2\text{-manifolds})$.

Teo: ogni $M^{(3)}$ si esprime come chirurgia intorno su \mathbb{S}^3 lungo un link con componenti banali.

Con: ogni $M^{(3)}$ è $\partial X^{(4)}$ X simplicemente connesso.

Ogni $M^{(3)}$ viene orientato.

Def: Heegaard splitting of M

• $\Sigma \subset M$ t.c. $M \setminus \Sigma = H_0 \cup H_1$, $H_0 \cong H_1 \cong U(\text{torus})$

• $M = H_0 \cup_f H_1$ H_0, H_1 handlebody
 $f: \partial H_1 \rightarrow \partial H_0$

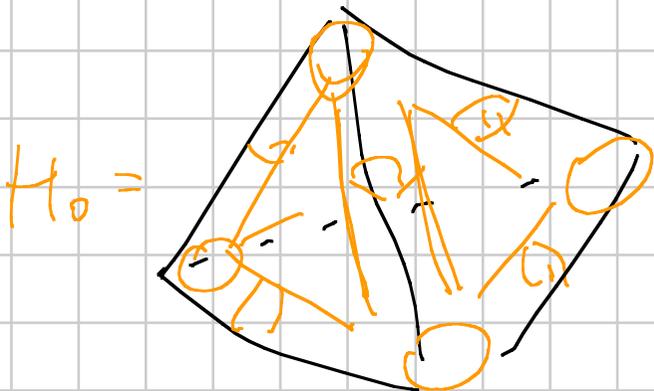


Prop: ogni $M^{(3)}$ ha Heegaard splitting.

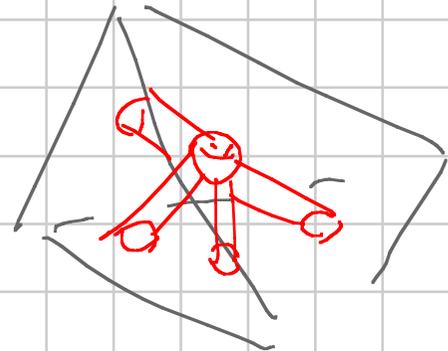
DM: DIFF. Note che

$$M^3 = \underbrace{(0\text{-manico}) \cup (1\text{-manico})}_{\text{handle body}} \cup \underbrace{(2\text{-manico}) \cup (3\text{-manico})}_{\text{handle body}}$$

TOP \cong tria $H_0 = \cup (| \mathbb{Z}^{(1)} |)$



$M \setminus H_0$



handle body - $= \cup$ (große Lücke)
 \Rightarrow handle body. ▣

Prop.: Seien H_0, H_1 handle body

$$g: \partial H_1 \rightarrow \Sigma, \quad f: \Sigma \rightarrow \partial H_0$$

$$\Rightarrow H_0 \cup_{f \circ g} H_1 = H_0 \cup_{f_0} (\Sigma \times [0,1]) \cup_{g_1} H_1$$

$$g_1: \partial H_1 \rightarrow \Sigma \times \{1\}$$

$$g_1(x) = (g(x), 1)$$

$$f_0: \Sigma \times \{0\} \rightarrow \partial H_0$$

$$f_0(x, 0) = f(x)$$

Div: $\tilde{H}_0 = H_0 \cup_i (\partial H_0 \times [0,1])$

$i: \partial H_0 \times \{0\} \rightarrow \partial H_0$ $i(x,0) = x$

$h: \partial H_1 \rightarrow \partial H_0 \times \{1\}$ $h(x) = ((f \circ g)(x), 1)$

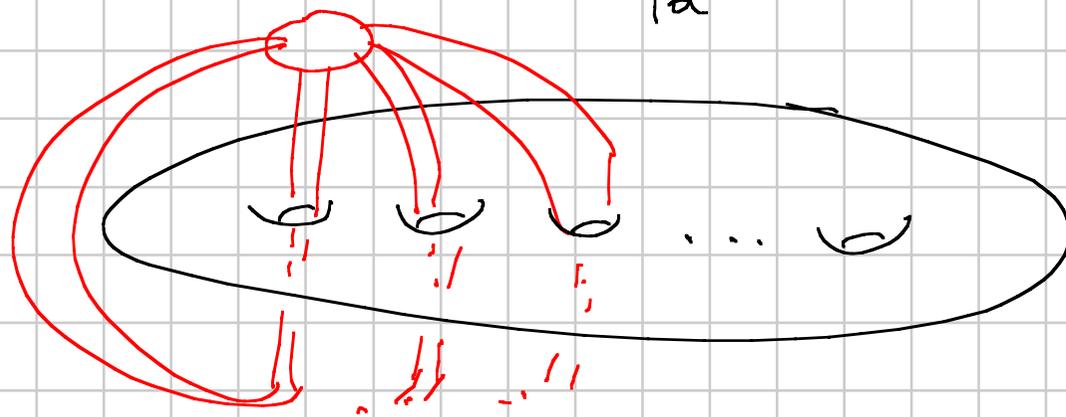
Barcode vedere das $\tilde{H}_0 \cup_h H_1 \cong H_0 \cup_{f_0} (\Sigma \times [0,1]) \cup_{g_1} H_1$.

$$\begin{array}{ccccccc}
 H_0 & \cup_{f_0} & (\Sigma \times [0,1]) & \cup_{g_1} & H_1 & & \\
 \downarrow \mu & & \downarrow (f(x), t) & & \downarrow \nu & & \\
 H_0 & \cup_i & (\partial H_0 \times [0,1]) & \cup_h & H_1 & &
 \end{array}$$

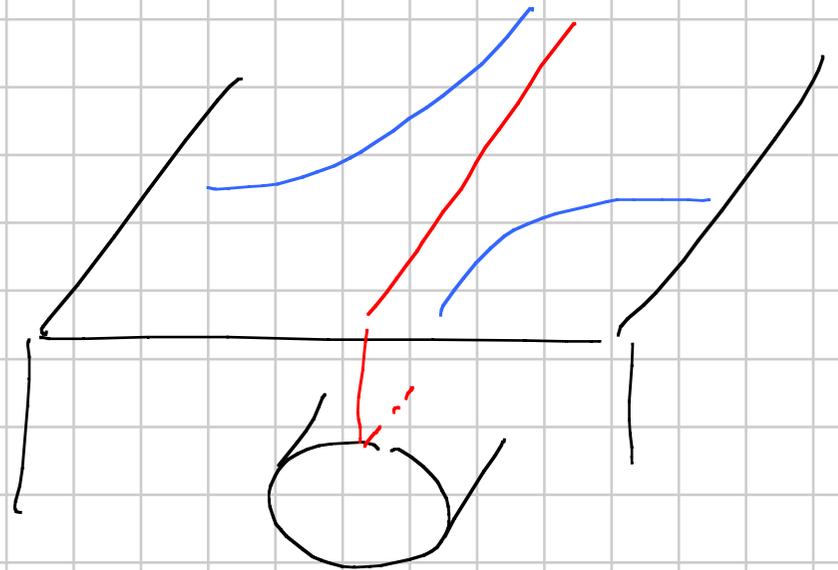
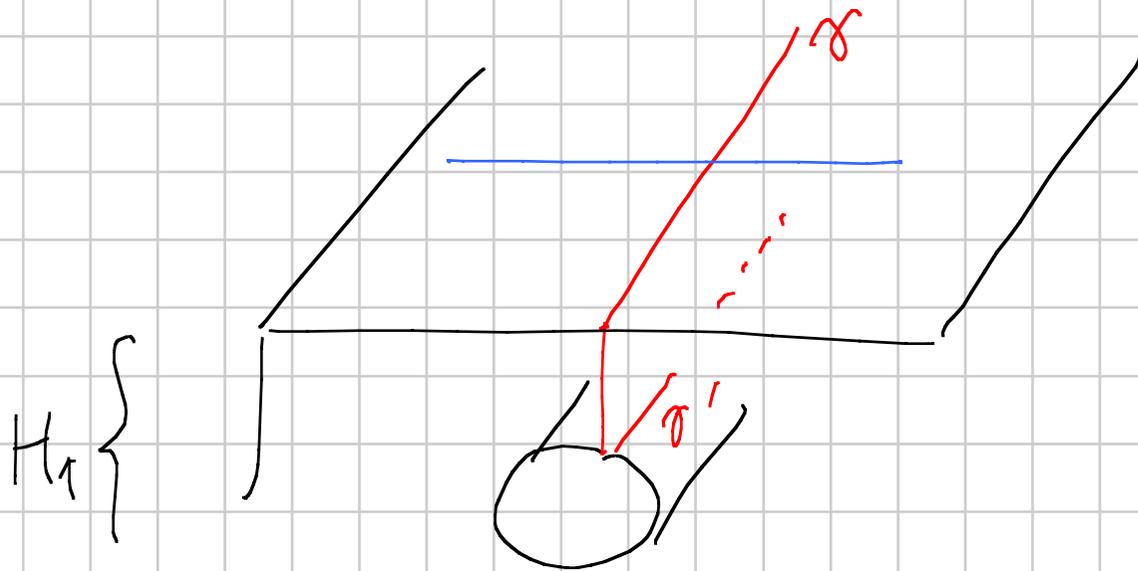
Esercizio: verificare che \tilde{i} ben def. \Rightarrow unico. \square

Dim (Teo): $H_0 U_f H_1$ si ottiene da S^3 per disingere
infine lungo link con componenti banali.

Punto da $S^3 = H_0 \cup_{id} H_1$



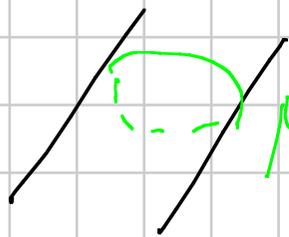
Case 1: $f: \mathbb{Z} \times \mathbb{R}$. Spingo γ dentro H_2 a γ'
 e considero $U = U(\gamma')$. Osservo che $\tau_\gamma: \partial H_1 \rightarrow \partial H_1$
 si estende a $f: H_1 \setminus U \hookrightarrow$



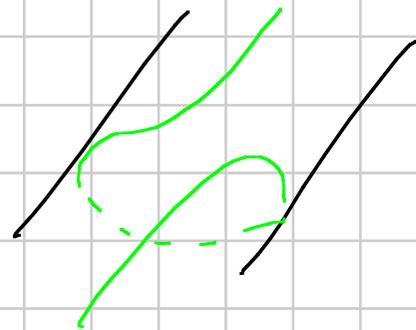
$$\Rightarrow M \setminus U = H_0 \cup_{\tau} (H_1 \setminus U)$$

$$S^3 \setminus U = H_0 \cup_{\text{id}} (H_1 \setminus U)$$

$\Rightarrow M$ è ottenuta da S^3 per chirurgia lungo nodo,
lungo le curve:



in H_1 in M



in S^3

$\Rightarrow \bar{e} \pm 1$ chirurgia

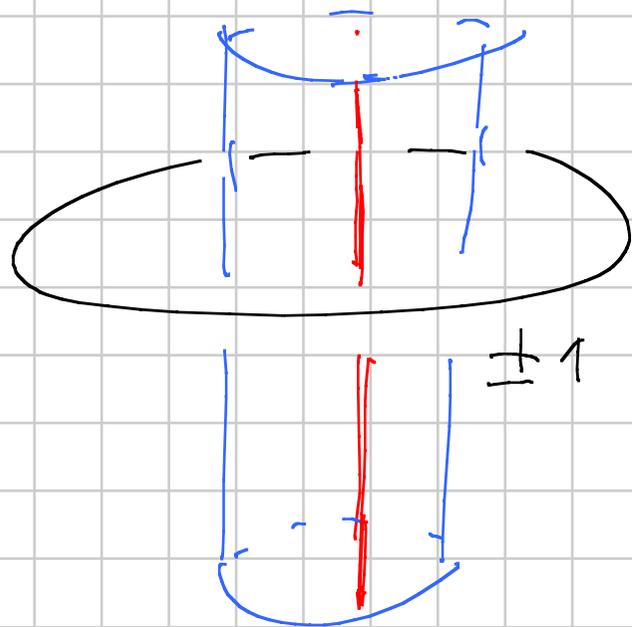
Se $f = \tau_{\alpha_0} \circ \dots \circ \tau_{\alpha_m}$

$$\begin{array}{ccccccc} H_0 \cup_{\tau_{\alpha_0}} (\Sigma \times [0,1] \setminus U_0) & \cup_{\tau_{\alpha_1}} & (\Sigma \times [0,1] \setminus U_1) & \cup \dots & & & \cup (H_1 \setminus U_m) \\ & \downarrow f_0 & & \downarrow f_1 & - & - & \downarrow f_m \\ H_0 \cup_{id} (\Sigma \times [0,1] \setminus U_0) & \cup_{id} & (\Sigma \times [0,1] \setminus U_1) & \cup \dots & & & \cup (H_1 \setminus U_m) \end{array}$$

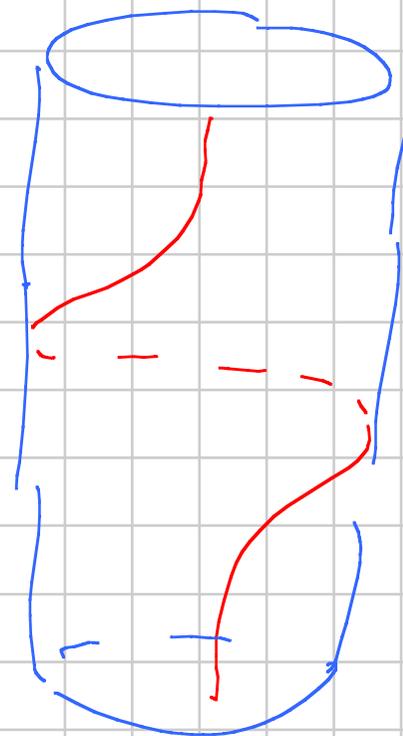
Some: M , tori solidi

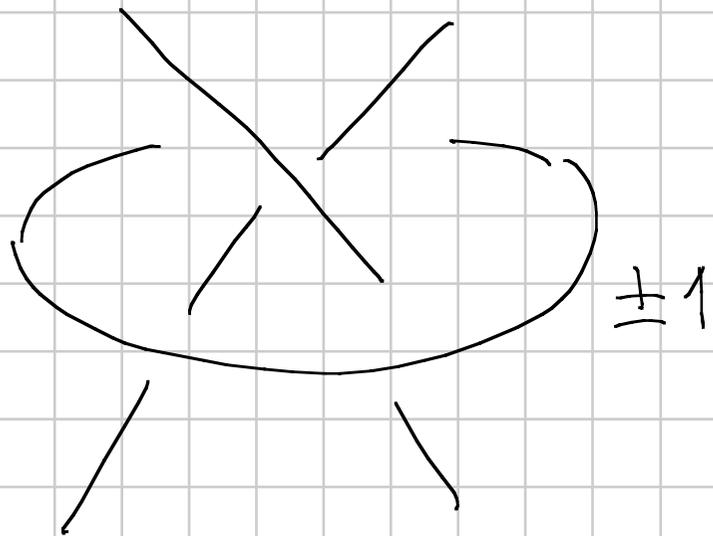
solito: $S^1 \times S^2$, tori solidi $\Rightarrow \bar{e}$ chirurgia con coeff ± 1 .

Per ottenere L con campo base:

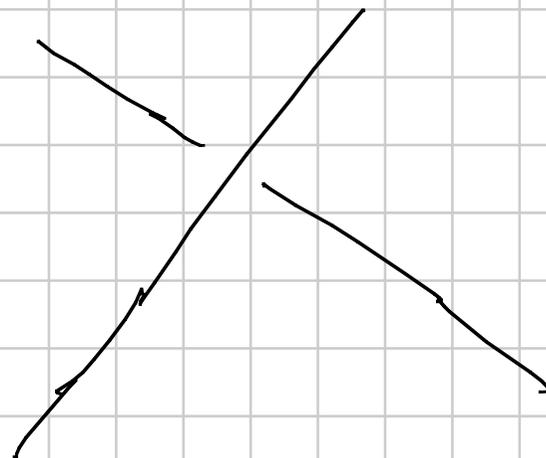


" \mathbb{Z}^3





=



e il coeff di chirurgia
variano di ± 1

□

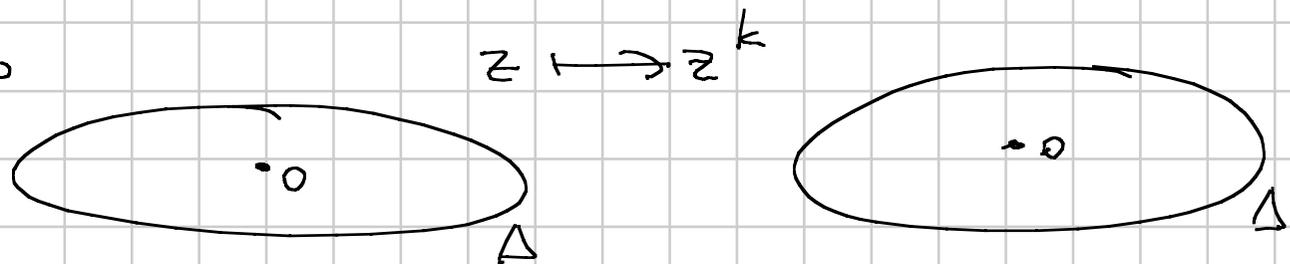
Rivertanti ramificati.

$$f: M^{(m)} \rightarrow N^{(m)} \quad \text{riv. ramif. } \alpha$$

esistono $A \subset M, B \subset N$ sottoinsiemi di codim. 2 t.c.

$$f(A) = B, \quad A = f^{-1}(B) \quad \text{e} \quad f|_{M \setminus A}: M \setminus A \rightarrow N \setminus B \quad \text{rivertamento.}$$

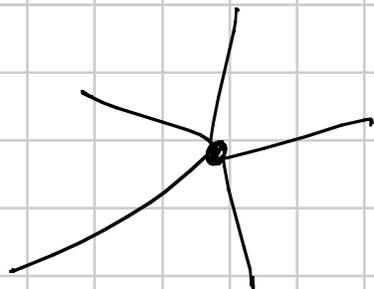
$m=2$: modello



$m=3$:



$$(z, t) \mapsto (z^k, t)$$



$\gamma_i = \text{complicato}$.

Prop: ogni $M^{(m)}$ chiusa orientata PL è riv. manif. di S^m
con insieme di riv. $|\Delta_m^{(m-2)}|$

Dim: $\sqrt{n=3}$.
esprimo S^3 con $\Delta_3 \cup_a \Delta_3$ cioè

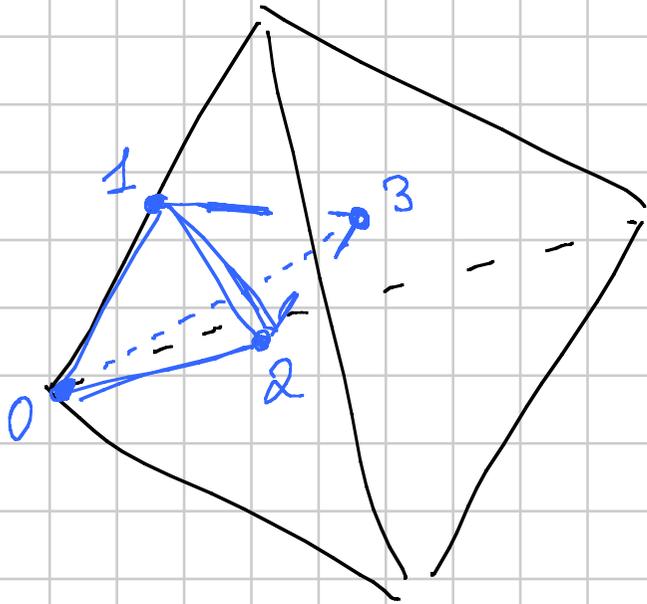
$$S^3 = f_+(\Delta_3) \cup f_-(\Delta_3) \quad f_{\pm} : \Delta_3 \rightarrow S^3$$

$$f_+ / a = f_- / a \quad \begin{array}{l} f_+ \text{ curva orientata} \\ f_- \text{ inverta.} \end{array}$$

Prendo \mathcal{T} triang. di M , \mathcal{T}' suddivisione baricentrica;

$$\Delta_3 = \text{Conv}(e_0, e_1, e_2, e_3). \quad \forall \sigma \in \mathcal{T}'^{(3)} \text{ prendo}$$

$p_{\sigma} : \sigma \Rightarrow \Delta_3$ che manda il baricentro di un j -simplex di \mathcal{T} in e_j



Definisco $f: M \rightarrow S^3$

come $f_+ = p_\sigma$ su σ t.c. 0123
 è orientaz. pos. di M

$f_- = p_\sigma$ su σ t.c. 0123
 è orientaz. neg.

Per costruzione $f|_{\text{int}(\sigma)}$

è omne locale

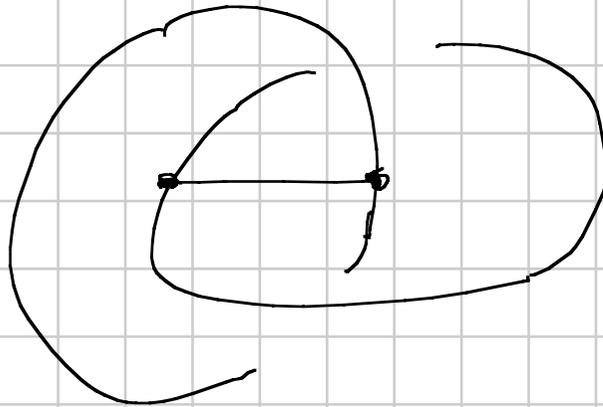
$\sigma \in \mathcal{O}'^{(3)}$

e anche $f|_{\text{int}(\eta)}$

$\eta \in \mathcal{O}'^{(2)}$

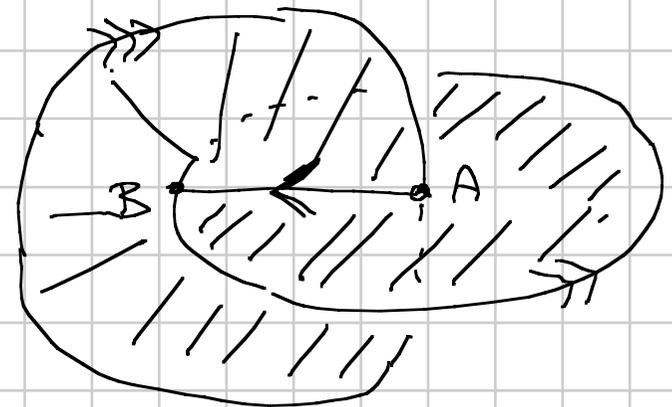
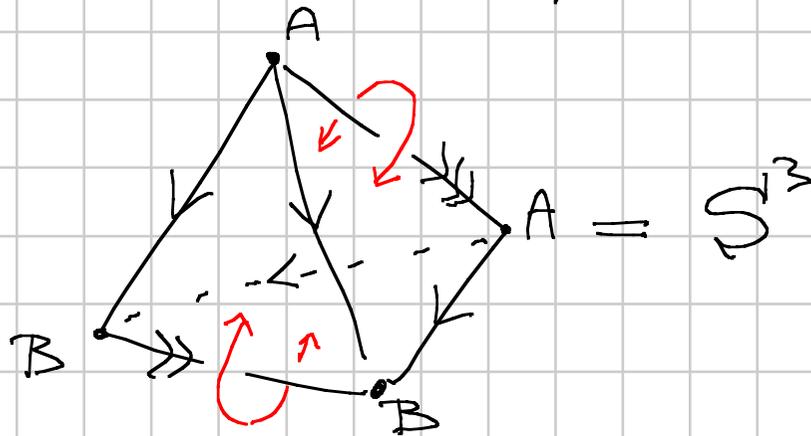
$\Rightarrow f|_{M \setminus \mathcal{O}'^{(1)}} \text{ riv.}$

Esercizio: anche



è insieme di surf. unir. per le 3-var chiuse e orientate.

Rapione



Teo (Hilden-Montenkos): $M^{(3)}$ chiuse orientate

$\Rightarrow \exists f: M \rightarrow S^3$ riv. ramif. di grado 3
ramificato lungo un nodo.

Lemma 1: $p_j: M_j \rightarrow N_j$ riv. ramif. tra varietà con ∂
e insieme di ramif. trasverso al ∂
 $j = 1, 2$

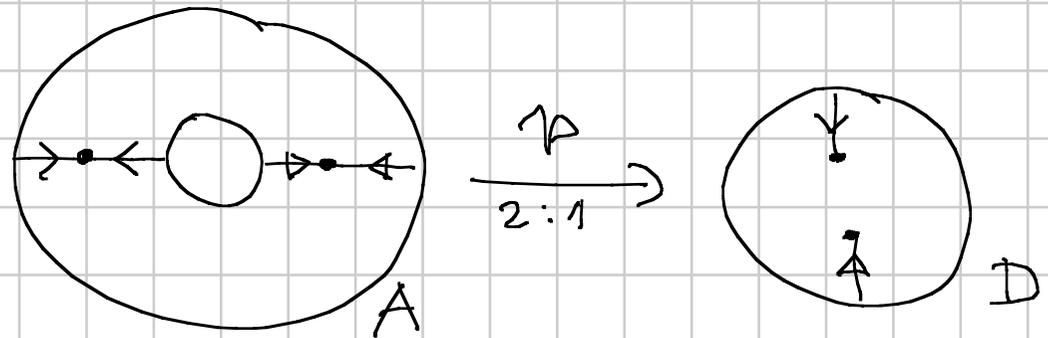
$$f: \partial M_1 \rightarrow \partial M_2 \quad g: \partial N_1 \rightarrow \partial N_2$$

$$p_2 \circ f = g \circ p_1$$

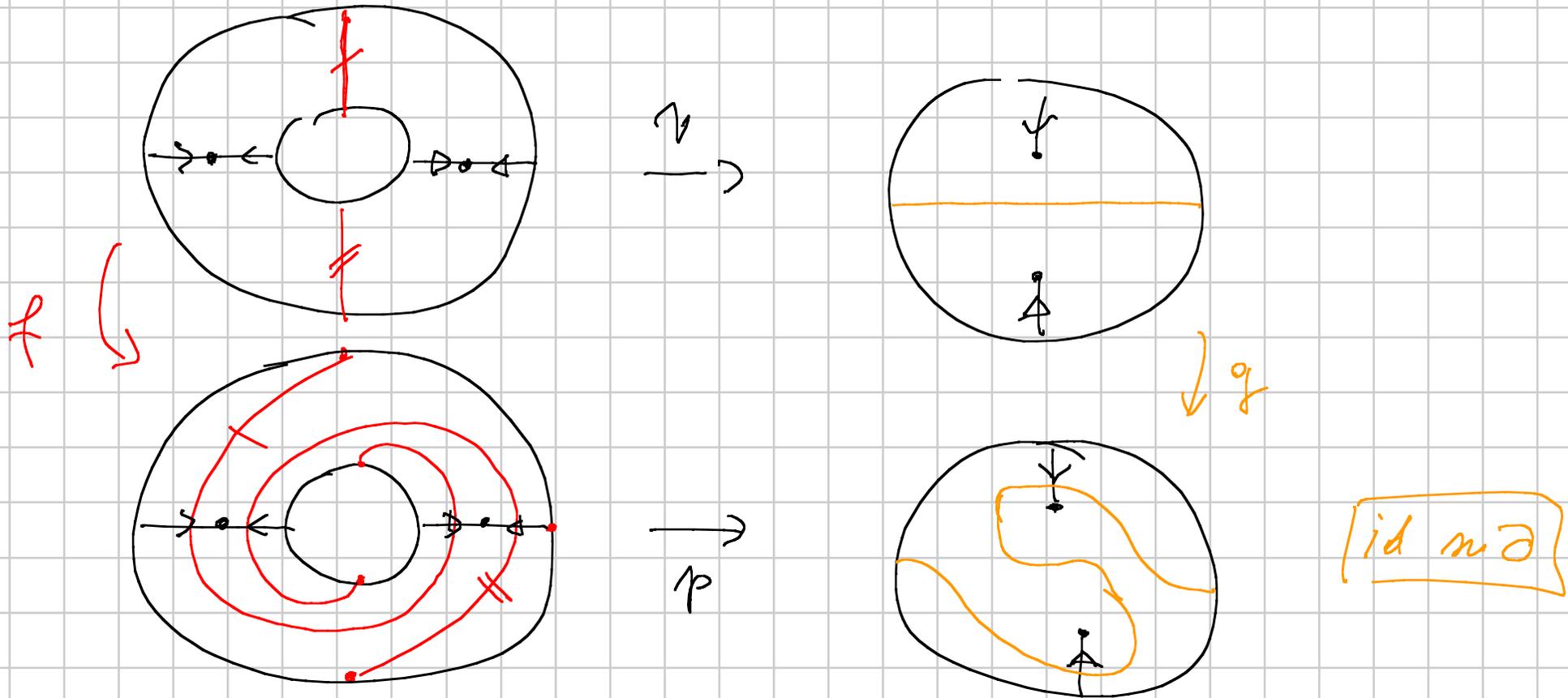
$$\begin{array}{ccc}
 \partial M_1 & \xrightarrow{f} & \partial M_2 \\
 p_1 \downarrow & & \downarrow p_2 \\
 \partial N_1 & \xrightarrow{g} & \partial N_2
 \end{array}
 \Rightarrow p: M_1 \cup_f M_2 \rightarrow N_1 \cup_g N_2$$

$p|_{M_1} = p_1, \quad p|_{M_2} = p_2$
 \bar{e} ein riv. Randfunkt.

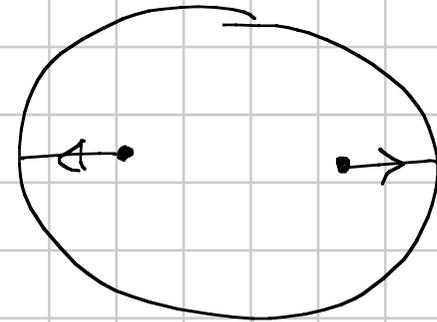
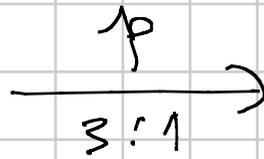
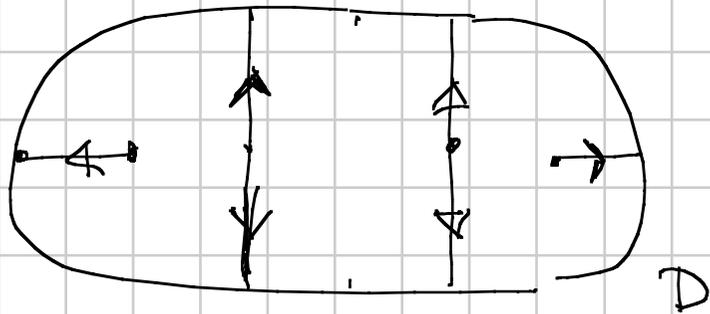
Lemma 2: Considerations



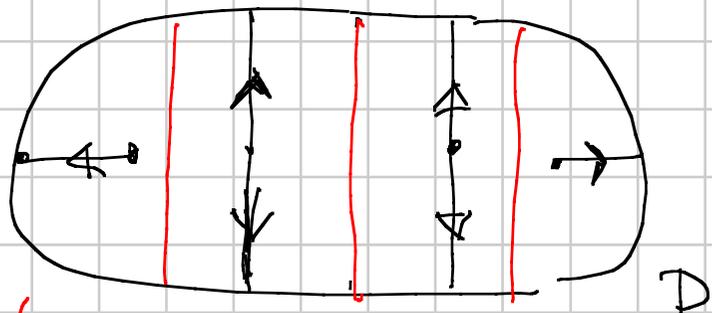
Allora i seguenti diagrammi compongono:



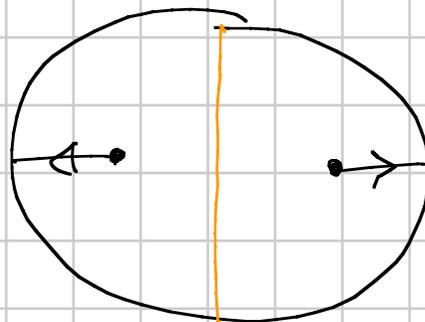
Lemma 3: Consideriamo



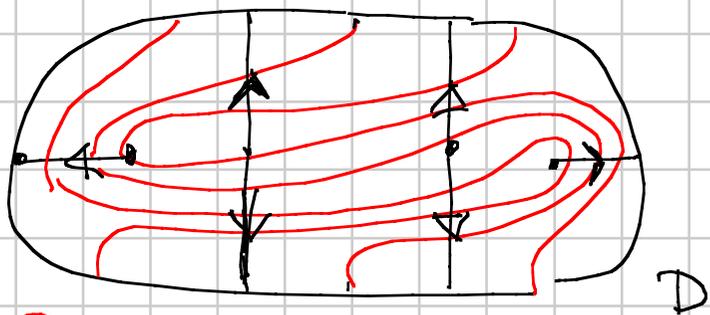
Allora il seguente diagramma commuta:



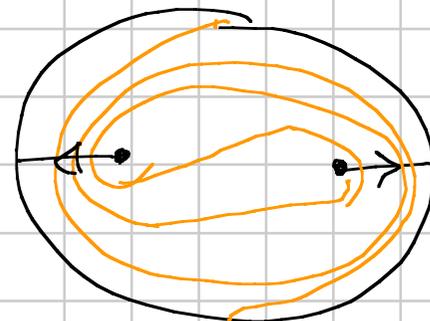
$$p \begin{matrix} \rightarrow \\ 3:1 \end{matrix}$$



f

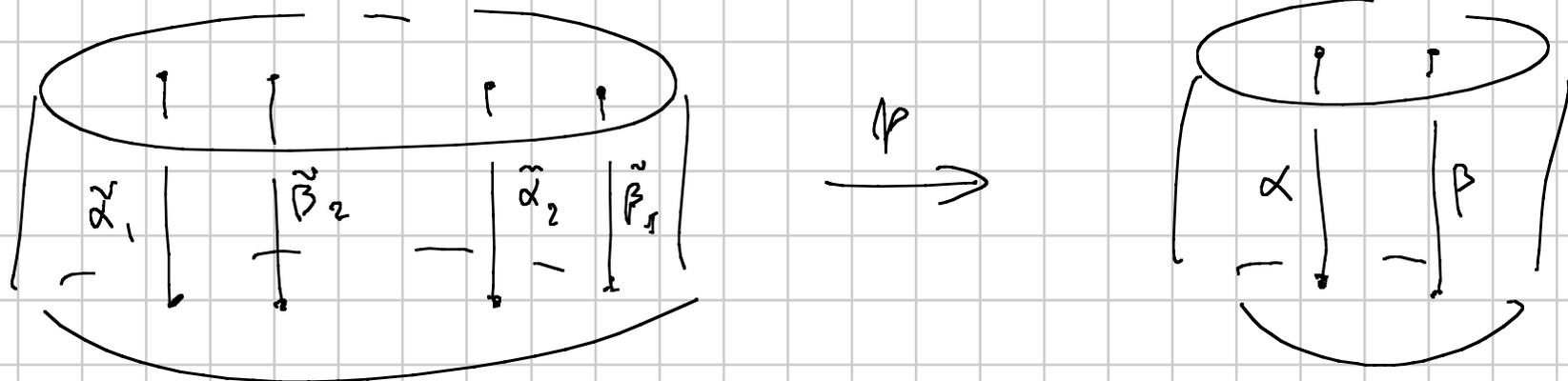


$$p \begin{matrix} \rightarrow \\ 3:1 \end{matrix}$$



id on ∂
 cut v : not 3π

Dim (cenni): Prendo (riv. Lem 3) $\times [0,1]$

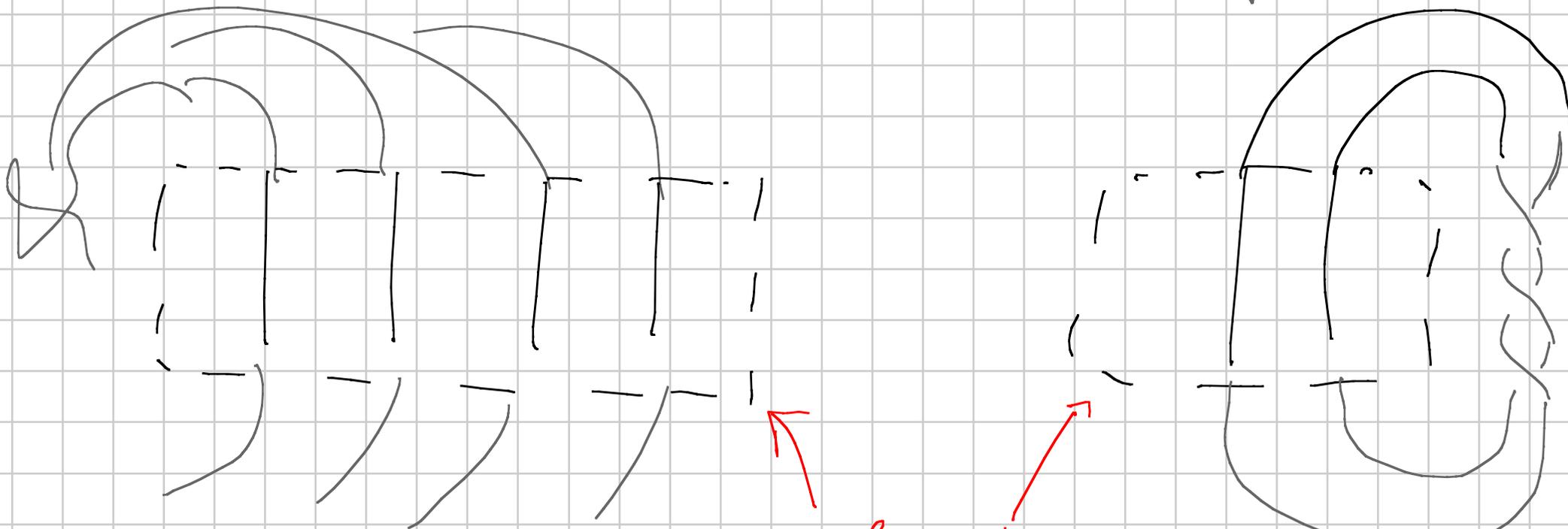


Ne attacco due copie lungo mappa che è la f/g
 applicando il lemma 1 tra $D^1 \times \{1\}$ e $D^1 \times \{1\}$, id
 stesso.

⇒ trova sub. manif.

$$S^3 \xrightarrow{3:1} \Sigma^3$$

manifesto lungo la \mathbb{Z}_3



nel ripunto uno
solo quarto pezzo.

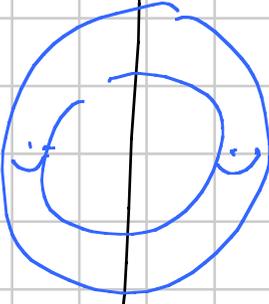
α_1



β_2



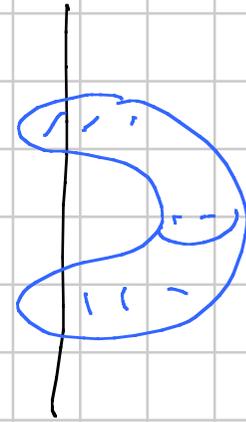
α_2



β_1



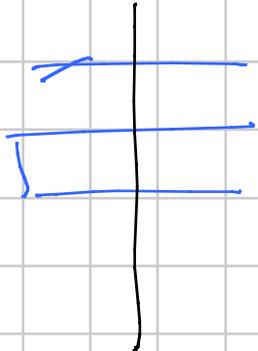
α



β

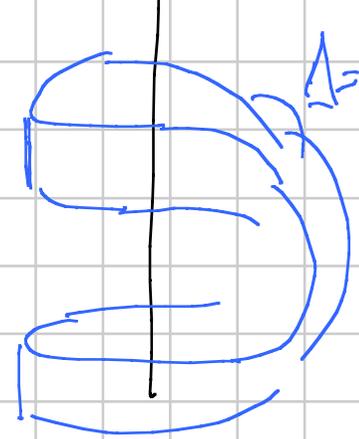


=



$A \times [0, 1]$

α



$A = \mathbb{D} \times [0, 1]$

Se vuoto Δ e lo riattacco con la mappa

$D^2 \times [0,1] \hookrightarrow S^3$ che è la g del Lem 2
su $D^2 \times \{1\}$ e id altrove

• Dopo un altro riv. canf. come da Lem 1 coprendo g
con id sul disco D^3 che incarta $\tilde{\alpha}_1$ e con

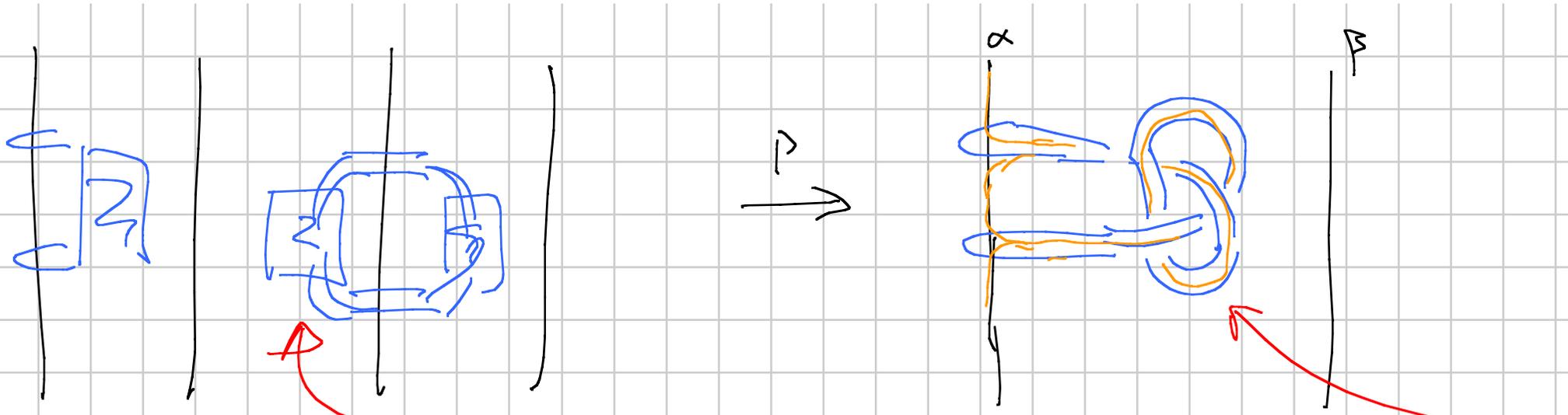
$$A \times [0,1] \rightarrow A \times [0,1]$$

su $A \times \{1\}$ è la f del Lem 2
altrove identico

Effetto di punto sull'immagine del riv. i. resta D^3

Sul dominio sto facendo una chirurgia lungo il toro solido,
anzi ± 1 chirurgia (dunque ancora S^1).

Posso poi applicare la stessa tecnica con embedding
iniziali di D^3 più complicati.



Ho trovato: le ± 1 dunque lungo α è un niv. caraf. di S^3 lungo il nodo

Per concludere devo provare che posso trasformare una

presentazione per chiurpie di quazioni M^2 in una
lungo link o demot. in questo modo.

