4/5/2018

11.2.2 Classificare a meno di trasf affini le coniche:

(a) $3x^2+6xy+5y^2-2xz-4yz-2z^2=0$

$$A=\begin{pmatrix} 3 & 3 & -1 \\ 3 & 5 & -2 \\ -1 & -2 & -2 \end{pmatrix}$$

per vedere se la conica è degenere calcolo $\det A$

$$\det(A) = 3(-10-4) - 3(-6-2) - 1(-6+5) =$$

$$= -42 + 24 + 1 = -17 \neq 0$$

$\Rightarrow$ conica non è degenere.

$$\alpha_2 = \det \begin{pmatrix} 3 & 3 \\ 3 & 5 \end{pmatrix} = 15 - 9 = 6$$

$\alpha_2 \neq 0 \Rightarrow$ non è una parabola.

$\alpha_1 = 3$, $\alpha_2 = 6$ $\Delta_3 = \det(A) < 0$

In coord. affini appartiene la conica

Una matrice $\begin{pmatrix} 1 & 0 & \\ 0 & 1 & \\ & & -1 \end{pmatrix}$ $\Rightarrow$ ellisse.

(e) $23x^2 - 8xy + 17y^2 - 75 = 0$

$$\begin{pmatrix} 23 & -4 & 0 \\ -4 & 17 & 0 \\ 0 & 0 & -75 \end{pmatrix} = A$$

$\det(A) = -75 \cdot (23 \cdot 17 - 16) < 0$

$\Rightarrow$ non degenere.

$$d_2 = (23 \cdot 17 - 16) > 0 \qquad , d_1 > 0$$

$\Rightarrow$ ellisse !

(f) $\quad 2x^2 - 3xy + y^2 - 7x + 6y + 5 = 0$

$$A = \begin{pmatrix} 2 & -\frac{3}{2} & -\frac{7}{2} \\ -\frac{3}{2} & 1 & 3 \\ -\frac{7}{2} & 3 & 5 \end{pmatrix}$$

$$\det(A) = 2\left(5 - 9\right) + \frac{3}{2}\left(-\frac{15}{2} + \frac{21}{2}\right) - \frac{7}{2}\left(-\frac{9}{2} + \frac{7}{2}\right) =$$

$$= -8 + \frac{9}{2} + \frac{7}{2} = 0$$

$\Rightarrow$ la conica è degenere.

$$\left[y^2 + y(-3x+6) + \left(3-\frac{3}{2}x\right)^2\right] - \left(3-\frac{3}{2}x\right)^2 - 7x + 5 + 2x^2 =$$

$$= \left(y - \frac{3}{2}x + 3\right)^2 - \frac{1}{4}x^2 + 2x - 4 =$$

$$= \left(y - \frac{3}{2}x + 3\right)^2 - \frac{1}{4}\left(x^2 - 8x + 16\right) =$$

$$= \left(y - \frac{3}{2}x + 3\right)^2 - \left[\frac{1}{2}(x-4)\right]^2 =$$

$$= (y - x - 4)(y - 2x + 4)$$

2 rette non parallele.

(g) $x^2+4xy+y^2+2x-2y-3=0$

$$A=\begin{pmatrix}1 & 2 & 1\\ 2 & 1 & -1\\ 1 & -1 & -3\end{pmatrix}$$

$$\det(A)=1(-3-1)-2(-6+1)+(-2-1)=$$

$$=-4+10-3=3>0$$

$\Rightarrow$ conica non degenere.

$d_2=1-4=-3$ $\Rightarrow$ iperbole.

(h) $4x^2+2xy+y^2+2x+4y=0$

$$A = \begin{pmatrix} 4 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 0 \end{pmatrix}$$

$$\det(A) = 2 - 1 - 2(8-1)$$
$$= 1 - 14 = -13 < 0$$

conica non degenere.

$$d_2 = 4 - 1 = 3 \quad , \quad d_1 > 0 \quad \Rightarrow \text{ellisse}.$$

(i)

$$4x^2 - 3y^2 + 8x + 12y + 4 = 0$$

$$\begin{pmatrix} 4 & 0 & 4 \\ 0 & -3 & 6 \\ 4 & 6 & 4 \end{pmatrix} = A$$

$$\det A = 2 \cdot 4 \cdot 3 \cdot \det \begin{pmatrix} 1 & 0 & 1 \\ 0 & -1 & 2 \\ 2 & 3 & 2 \end{pmatrix} = 24 \cdot (-8+2) = -6 \cdot 24 < 0$$

$\Rightarrow$ conica non degenere. $\quad d_2 = -12 < 0$

$\Rightarrow$ iperbole.

(k) $\quad 2x^2 + 6xy + 5y^2 + 4x + 10y + 10 = 0$

$$A = \begin{pmatrix} 2 & 3 & 2 \\ 3 & 5 & 5 \\ 2 & 5 & 10 \end{pmatrix}$$

$$\det(A) = 2 \cdot 5 (10 - 5) - 3 (30 - 10) + 10 (3 - 2) =$$

$$= 50 - 60 + 10 = 0$$

$\Rightarrow$ conica degenere

$$2\left[x^2 + x(3y + 2) + \left(\frac{3}{2} y + 1\right)^2 \right] - 2\left(\frac{3}{2} y + 1\right)^2 + 5y^2 + 10y + 10 =$$

$$= 2\left(x + \frac{3}{2} y + 1 \right)^2 + \frac{1}{2} y^2 + 4y + 8 =$$

$$= 2\left(x + \frac{3}{2}y + 1\right)^2 + \frac{1}{2}\left(y^2 + 8y + 16\right) =$$

$$= 2\left(x + \frac{3}{2}y + 1\right)^2 + \frac{1}{2}\left(y + 4\right)^2$$

la conica è un punto

$$(5, -4)$$

(P) $\quad x^2 + 2xy + y^2 + \sqrt{2}\,x - \sqrt{2}\,y + 2 = 0$

$$\begin{pmatrix} 1 & 1 & \frac{1}{\sqrt{2}} \\ 1 & 1 & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 2 \end{pmatrix} = A$$

$$\det(A) = \det \begin{pmatrix} 0 & 1 & \frac{1}{\sqrt{2}} \\ 0 & 1 & -\frac{1}{\sqrt{2}} \\ \sqrt{2} & -\frac{1}{\sqrt{2}} & 2 \end{pmatrix} = -\sqrt{2}\left(\frac{2}{\sqrt{2}}\right) = -2 \neq 0$$

conica è non degenere

$d_2 = 0 \quad \Rightarrow$ parabola